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ENVIRONMENTAL ENGINEERING PROGRAM
DEPARTMENT OF CIVIL ENGINEERING
UNIVERSITY OF MASSACHUSETTS
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ABSTRACT

Field testing and application of the mathematical model developed by Snow and DiGiano in 1976 for prediction of recovery of shallow eutrophic lakes is described in this report. A field measurement program was carried out to observe the effect of a reduction in phosphorus loading to Lake Warner in Hadley, Massachusetts.

It was found that Lake Warner had recovered, as indicated by the reduction of the lake water phosphorus concentration, at a faster pace than the prediction by the Snow and DiGiano model. This discrepancy could be the result of the assumption made by Snow and DiGiano that the linear empirical relationship between the solid-phase and liquid-phase phosphorus of sediment developed by them would remain unchanged during the recovery of Lake Warner.

It is proposed to modify the Snow and DiGiano model by replacing the empirical expression by a mass balance equation on the sediment phosphorus. A new reaction rate is introduced to describe the net rate of conversion between solid-phase and liquid-phase phosphorus of sediments. Although this reaction rate for Lake Warner was not determined experimentally in the study, the modified model was tested by using the conversion rate constants determined by other researchers.

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NOMENCLATURE

Symbol	Definition and Units in MLT System
A	Lake Surface Area (L^2)
A_c	Stream Cross-sectional Area (L^2)
a	Stage-Discharge Relation Coefficient
B	Stream Channel Width (L)
b	Ordinate Coefficient in Linear Sediment Relationship $P_i = mP_s+b$
c	Stage-Discharge Relation Coefficient
d_r	Sediment Reactive Depth (L)
h	Stream Channel Depth (L)
i	Number of Measured Data Points
J	Loading of Substance to Lake Water (M/T)
k_1	Release Rate Constant (L/T)
k_2	Sedimentation Rate Constant (1/T)
k_3	Fraction of Phosphorus Input to Sediment Unavailable for Exchange
K_1	Release Rate Constant (L/T)
K_2	Sedimentation Rate Constant (L/T)
K_3	Phosphorus Desorption Coefficient (1/T)
L	Lake Phosphorus Loading (M/L^2T)
M	Total Annual Phospohrus Loading (M/T)
m	Slope Coefficient in Linear Sediment Relationship $P_i=mP_s+b$
m_w	Mass of Substance in Lake Water (M)
n	Manning's Coefficient ($T/L^{1/3}$)
P	Phosphorus Concentration (M/L^3)
P_i	Interstitial Sediment Phosphorus Concentration (M/L^3)

Symbol	Definition and Units in MLT System
P_{in}	Measured Influent Phosphorus Concentration (M/L^3)
P_L	Lake Water Phosphorus Concentration (M/L^3)
P_{lo}	Initial Lake Water Phosphorus Concentration (M/L^3)
P_o	Influent Phosphorus Concentration (M/L^3)
P_{out}	Measured Effluent Phosphorus Concentration (M/L^3)
P_s	Dry Weight Sediment Phosphorus Content (M/M)
P_t	Percent Exchangeable Phosphorus
p	Phosphorus Concentration (M/L^3)
p_a	Admissible Phosphorus Concentration (M/L^3)
p_d	Dangerous Phospohrus Concentration (M/L^3)
Q	Outflow Volume per Year (L^3/T)
Q_{in}	Measured Lake Inflow (L^3/T)
Q_o	Lake Inflow and Outflow (L^3/T)
Q_{out}	Measured Lake Outflow (L^3/T)
R	Hydraulic Radius (L)
R_x	Reaction Rate (1/T)
S	Channel Slope (L/L)
$\bar{T}$	Lake Detention Time (T)
t	Time (T)
t_{rel}	Ratio of Release Rate to Sedimentation Rate (Dimensionless)
$\bar{t}_{H_2O}$	Mean Hydraulic Residence Time (T)
$\bar{t}_p$	Mean Phosphorus Residence Time (T)
V	Lake Volume (L^3)

Symbol	Definition and Units in MLT System
V_L	Lake Volume (L^3)
V_s	Sediment Volume (L^3)
$\bar{z}$	Mean Depth of Lake (L)
ε	Porosity (Dimensionless)
σ	Sedimentation Rate Constant (1/T)

CHAPTER I

INTRODUCTION

The increased activities of man in recent decades have greatly enhanced the inflow of nutrients to many freshwater lakes which results in accelerated growth of algae and other plants. The growth and decay of these plants pollute lakes and inhibit their recreational uses. This condition, known as eutrophication, has become a severe problem in many of our freshweater lakes.

Management plans are underway in many locations to alleviate the eutrophication problem. Alternative methods of controlling nutrient inflows must be evaluated as to their effectiveness. The engineer or planner needs reliable predictive tools such as mathematical models to develop cost-effective management plans. This research is to test one such mathematical model.

The mathematical model to be tested was developed by Snow and DiGiano (1976) for the prediction of lake recovery from a eutrophic state after a reduction in phosphorus loading. The model was developed on the basis of the sources, sinks, and reactions involved in the phosphorus cycle of Lake Warner. Hydrodynamic parameters such as residence time and flow rate were measured, and reaction rates were determined by <u>in situ</u> tests. An important prediction made by Snow and DiGiano based on their model was that the recovery of Lake Warner to natural phosphorus concentrations would take about 14 years. The long period of time required for lake recovery was attributed to the slow release of phosphorus from the lake sediments. This research attempts to test the model with a field study program in two aspects: (1) whether the lake is recovering as predicted, and (2) if not, how should the model be modified.

This report begins with a literature review on the mathematical modeling of eutrophication, emphasizing phosphorus dynamics, followed by a description of Lake Warner, a field measurement program designed to test the model, and the data thus collected. The data are then analyzed in regard to the state of eutrophication of Lake Warner, and compared with the model predictions.

CHAPTER II

LITERATURE REVIEW

The following is a review of mathematical modeling for the prediction of phosphorus cycling in fresh water lakes. Of particular importance is the rate of recovery of a eutrophic lake after a reduction of phosphorus inflow. Such models can be a valuable tool for planning the strategy of controlling phosphorus.

Empirical Models

Models in existence today range from empirical procedures relating phosphorus loading estimates and lake hydraulic parameters to more sophisticated models derived from mass balances on phosphorus cycling in lakes. The first of the empirical models to gain wide acceptance was the phosphorus loading-mean depth relationship developed by Vollenweider (1968). This model has been used as a guide for determining the degree of eutrophy or "trophic state" of lakes and to estimate "admissible" and "dangerous" loading levels. Admissible loadings allow the lake to remain oligotrophic and dangerous loadings cause the lake to become eutrophic. Based on many lake water quality surveys, the following two equations were found to define rough boundaries between eutrophic, mesotrophic, and oligotrophic conditions:

$$\log P_a = 0.60 \log \bar{z} + 1.40$$

$$\log P_d = 0.60 \log \bar{z} + 1.70$$

where:

P_a = admissible phosphorus loading ($g/m^2/yr$)

P_d = dangerous phosphorus loading ($g/m^2/yr$)

$\bar{z}$ = mean depth of lake (m)

Vollenwider (1975) later modified his model to include the mean hydraulic residence time. In this relationship, the allowable loading increases as the depth increases and the residence time decreases. The residence time is the inverse of the flushing frequency, or number of times per year the lake water is replaced. Therefore, generally speaking, the more replacement of lake water that occurs, the more loading the lake can tolerate.

Dillon (1974) also realized the importance of the flushing rate in lake eutrophication. His version of the trophic state model included the flushing rate and the lake retention coefficient. The retention coefficient is that fraction of phosphorus not lost through the outflow, or, the portion of the phosphorus subject to sedimentation. This concept takes into account the fact that phosphorus does not act as a conservative substance in a lake. Dillon (1974) used a mass balance developed by Vollenweider (1969) for nutrients and written as follows:

$$\frac{dm_w}{dt} = J - \sigma m_w - \frac{Q}{V} m_w$$

where:

m_w = mass of substance in lake water (kg)

J = loading of substance to lake water (kg/yr)

σ = sedimentation rate coefficient (yr^{-1})

Q = outflow volume per year (m^3/yr)

V = lake volume (m^3)

The steady-state solution to this equation for phosphorus is:

$$P = \frac{L}{\bar{z}\ (\sigma + p)}$$

where:

P = phosphorus concentration (mg/m^3)

L = lake phosphorus loading ($mg/m^2/yr$)

p = flushing rate (yr^{-1}) = Q/V

$\bar{z}$ = mean depth of lake (m)

This model assumes a constant loading through time, a completely mixed lake, and first-order sedimentation. The retention coefficient R is defined as:

$$R = \frac{\sigma}{\sigma + p}$$

and by substituting:

$$P = \frac{L\ (1-R)}{\bar{z}\ p}$$

Dillon (1974) plotted log [L(1-R)/p] vs. log ($\bar{z}$), as shown in Figure 1. Lines of constant concentrations of 10 and

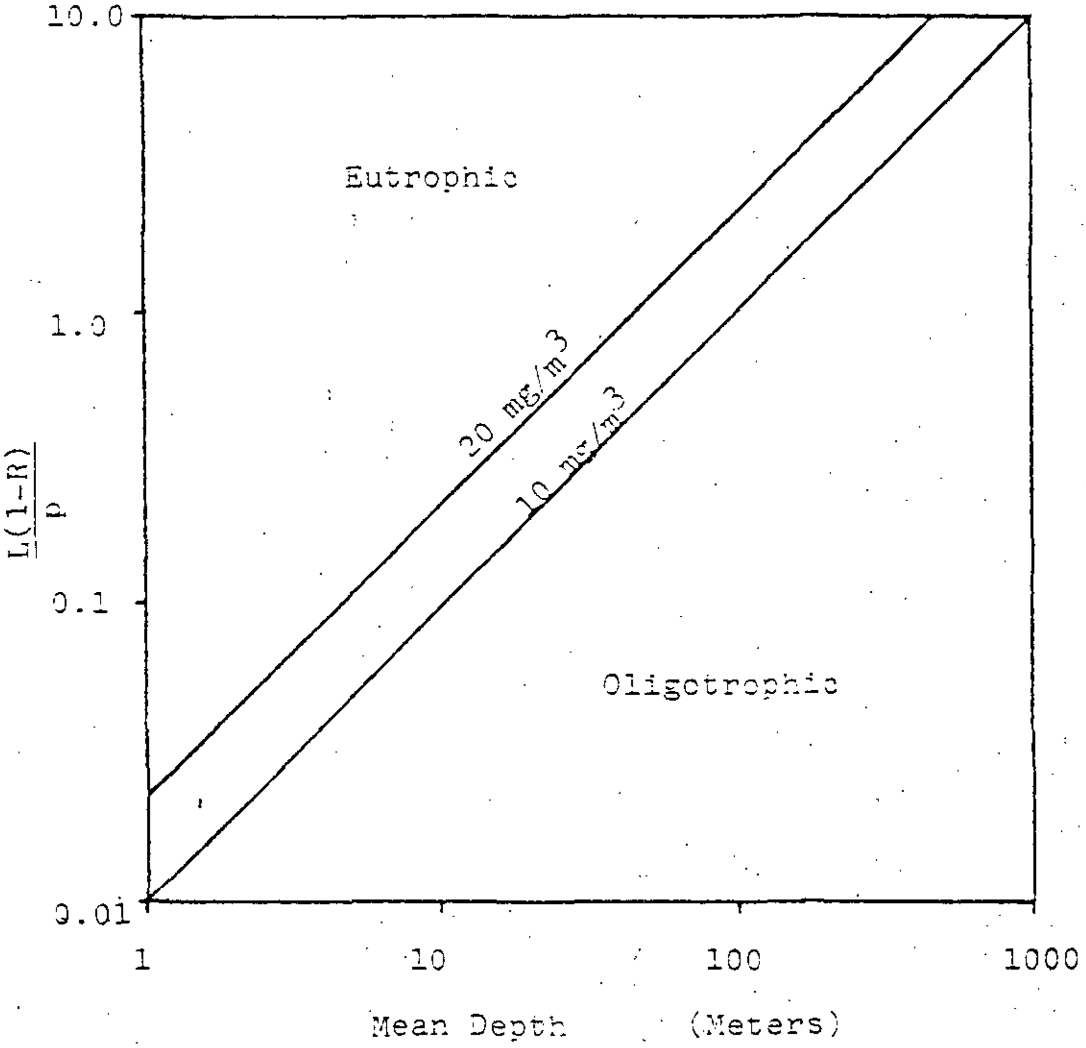

FIGURE 1.

PHOSPHORUS LOADING (Including Sedimentation and Flushing Rate) VS. MEAN DEPTH (after Dillon 1974)

20 mg/m^3 separated eutrophic, mesotrophic, and oligotrophic conditions.

The models reviewed to this point are simple, somewhat empirical methods which are useful in evaluating the existing state of eutrophication in a lake and some of the factors which affect that state. Since they are steady-state models, no information can be obtained on the time required for a lake to respond to a change in loading. This response time is critical in evaluating management plans designed to improve the eutrophic status of a lake. The empirical models do, however, point out the importance of lake depth, flushing time, and sedimentation rate in lake phosphorus content.

Dilution Models

If phosphorus acted as a conservative substance in a lake, the reaction rates could be eliminated from the mass balance and the following solution obtained (Ahlgren, 1977):

$$P_l = P_o + (P_{lo} - P_o) \exp\left(-\frac{Qt}{V}\right)$$

where:

P_l = lake phosphorus (M/L^3)

P_o = influent phosphorus concentration (M/L^3)

P_{lo} = lake phosphorus concentration at time $t = 0$ (M/L^3)

This simple dilution model indicates that a 95 percent reduction of the difference between the initial lake phosphorus concentration (P_{lo}) and the influent phosphorus concentration (P_o) would occur in about three residence times. It has already been indicated, however, that phosphorus is subject to sedimentation. This reaction would tend to accelerate the reduction in lake phosphorus concentration, as the lake sediment acts as a phosphorus sink.

Phosphorus may also be released from the sediment back into the lake water under favorable conditions. This release would tend to slow the reduction in lake phosphorus as predicted by the dilution model. Snow and DiGiano (1976), Lorenzen et al. (1976) and others have developed models which include both sedimentation and release of sediment phosphorus.

O'Melia (1972) has distinguished the difference between the mean residence times of water ($\bar{t}_{H_2O}$) and phosphorus ($\bar{t}_p$):

$$\bar{t}_{H_2O} = \frac{V}{Q} = \frac{\text{mass of water in lake}}{\text{mass flow rate of water into lake}}$$

$$\bar{t}_p = \frac{P}{dP/dt} = \frac{\text{mass of phosphorus in lake}}{\text{mass flow rate of phosphorus into lake}}$$

If total phosphorus acts conservatively, $\bar{t}_{H_2O} = \bar{t}_p$. If phosphorus is removed from the lake by a process such as sedimentation, $\bar{t}_p$ will be less than $\bar{t}_{H_2O}$. In the latter case, the lake water phosphorus will decrease more rapidly.

Ahlgren (1977) has monitored the recovery of Lake Norrviken since all wastewater effluents were removed in 1971. A dilution model of the following form was found to fit the data on a yearly basis:

$$P_1 = P_o + (P_{1o} - P_o) \exp\left(- \frac{Qt}{Vt_{rel}}\right)$$

where t_{rel} is a dimensionless factor expressing the ratio of release rate to sedimentation rate and $\frac{\bar{t}_{H_2O}}{t_{rel}}$ is the effective phosphorus residence time. For a t_{rel} larger than 1, release is greater than sedimentation. This causes a slower change in lake phosphorus concentration when progressing from higher to lower P values in time. During Ahlgren's study, a t_{rel} of 1.3 was found to cause a best fit to measured data. This indicates that phosphorus cycling processes such as sediment release caused the phosphorus to remain in the lake 1.3 times longer than the hydraulic residence time. The seasonal phosphorus concentrations oscillated around the dilution curve, with rapid sedimentation in spring after the diatom maximum causing low total P and summer release causing high total P.

Sedimentation and Release Models

A rational engineering approach to eutrophication modeling should attempt to take all major factors into consideration. Since sedimentation and release are both known to occur, models which include them will be reviewed here in detail. To aid the reader in the discussion of unsteady-state phosphorus models useful in evaluating lake response times, the following generalized mass balance is given:

$$V \frac{dP_L}{dt} = Q_o P_o - Q_o P_L \pm R_x P_L V$$

where:

P_L = lake phosphorus concentration (M/L^3)

P_o = influent phosphorus concentration (M/L^3)

Q_o = flow in and out of lake (L^3/T)

V = lake volume (L^3)

R_x = rates of reaction $(1/T)$

t = time (T)

This equation is derived from the assumption that the lake behaves as a complete-mixed reactor. This assumption is valid if there are no significant phosphorus gradients in the lake. Short-term transients may exist due to storm events, for example, but descriptions of these events are of little importance when studying eutrophication. Changes in lake eutrophic state usually happen over long time periods, often much greater than the hydraulic residence time of the lake, and therefore long-term changes are of interest. Other assumptions of this equation are that reaction rates are first order, and that flow is constant through the lake.

Lorenzen et al. (1976) modeled the recovery of Lake Washington after sewage diversion. The equations included sedimentation to the sediment, release from the sediment, and accounted for a fraction of the sediment phosphorus being unavailable for exchange. Mass balances were written for both the lake water and sediment as follows:

$$V \frac{dP_L}{dt} = M - Q_o P_L - k_1 A P_L + k_2 A P_s$$

$$V_s \frac{dP_s}{dt} = k_1 A P_L - k_2 A P_s - k_1 k_3 A P_L$$

where:

P_L = lake phosphorus concentration (M/L^3)

V = Volume (L^3)

P_s = sediment phosphorus concentration (M/L^3)

k_1 = sedimentation rate constant (L/T)

k_2 = release rate constant (L/T)

k_3 = fraction of phosphorus input to sediment that is unavailable for exchange (dimensionless)

M = total annual phosphorus loading (M/T)

V_s = volume of sediment (L^3)

A = surface area (L^2)

According to this formulation, a certain constant portion of the lake loading settles, and a constant portion of the sediment load is released. A fraction of the sediment was not available for release, however, which was estimated to be 60 percent ($k_3 = 0.6$). This non-exchangeable fraction was attributed to the oxidizing condition of the bulk of Lake Washington water.

Although the model gave reasonable predictions for Lake Washington, it contains many simplifying assumptions which could limit its applicability. The real reactions which transform phosphorus in the sediment and release it to the lake water have not been described, but instead replaced with the gross assumption of an amount settled per year minus 60 percent. Since only yearly loading rates are known, no seasonal variations may be accounted for. Also, the 60 percent approximation is a rough estimate at best and is certainly subject to change on a year-to-year basis as sediment equilibriums shift. This casts any long-term future predictions in doubt.

Imboden (1974) also developed a model in which sediment exchange was considered. As in the work by Lorenzen, the release was not based on actual processes, but was characterized as a constant flux at the sediment-water interface.

One of the most complete attempts at mathematical modeling of phosphorus was the work done at the University of Michigan on White Lake, Michigan. The model was developed and calibrated for White Lake by Lung, Canale, and Freedman (1976). A subsequent paper by Lung and Canale (1977) contained projections of phosphorus levels under various alternatives.

White Lake was assumed to act as two completely mixed reactors, the epilimnion and the hypolimnion. The sediment was modeled as four layers, each 5 cm thick. Both particulate and dissolved forms of phosphorus were accounted for. Particulate phosphorus is the solid-phase phosphorus associated with algae or particles in the water column or the sediment. Phosphorus exists in dissolved form in the water column or in the interstitial water of the sediment. Mechanisms which accounted for concentration changes in the water column were vertical eddy diffusion, settling of particulate phosphorus, influent loading and flow, and reaction from dissolved to particulate phosphorus or vice versa. Photosynthesis in the epilimnion was indicated to be the main cause of dissolved P converting to particulate P, and the reverse reaction was mainly due to mineralization in the hypolimnion. The reactions were assumed to be first order.

At the interface between the sediment and water column, settling carried particulate phosphorus to the lake bottom and diffusion transported the phosphorus in dissolved form back into the lake water. Within the sediments, both particulate and dissolved P could settle, while dissolved interstitial P could diffuse upward. Particulate P could be converted to dissolved P by a first-order reaction and desorbed from the sediment. The desorption reaction coefficient of sediment phosphorus was shown to vary inversely with dissolved oxygen. Greater release occurred during the summer months when anaerobic conditions existed in the hypolimnion. The reaction slowed to zero in the winter, when aerobic conditions occurred.

It has been shown that the dissolved oxygen in the sediment-water environment has a major effect on the phosphorus transformations which occur there. For example, Fillos and Swanson (1976) performed laboratory experiments on Lake Mohegan sediments and found that for a P concentration under 1-2 mg/ℓ, the sediments adsorbed phosphorus in an aerobic system and released phosphorus in an anaerobic system. Above the high phosphorus concentration of 1-2 mg/ℓ, however, sediments adsorbed phosphorus from the overlying water for both aerobic and anaerobic conditions.

Ku and Feng (1975) have shown that Lake Warner sediment adsorption capacity increased with increasing redox potential. They tested the sediment at both high (+300 to +500 mv) and low (-200 to -300 mv) redox potentials and found the adsorption capacity to be extremely sensitive to this parameter. Reducing conditions tended to allow greater release of phosphorus. Also, it has been shown that increases in temperature or pH were favorable for greater phosphorus release.

The seasonal variation of the reaction coefficient in the sediment which converted particulate to dissolved phosphorus used by Lung, Canale, and Freedman (1976), therefore, probably results from several factors. Summer conditions of high temperature and low DO in the hypolimnion, coupled with algal activity which raises the pH, all tend to favor sediment phosphorus release. In the winter, low temperatures, elimination of algal activity, and aeration of the hypolimnion all tend to favor adsorption of phosphorus by the sediments.

Snodgrass and O'Melia (1975) developed a phosphorus model for lakes which included sedimentation of particulate phosphorus but no other sediment interactions. The model included seasonal variations in both the hypolimnion and the epilimnion for particulate and dissolved reactive (ortho) phosphorus. They cautioned that their model could only be applied to lakes with oxic hypolimnetic waters, since only settling of phosphorus to the sediments and not sediment release was accounted for. The implication was that anoxic conditions would be required for sediment release, although this was not explicitly stated in their publication.

A model developed by Snow and DiGiano (1976) includes first-order sedimentation, diffusion-type release, and separate mass balances for the lake water and sediment as follows:

$$V \frac{dP_L}{dt} = Q_o P_o - Q_o P_L + \varepsilon A K_1 (P_i - P_L) - K_2 P_L V \qquad (2\text{-}1)$$

$$V \frac{dP_s}{dt} = K_2 P_L V - \varepsilon A K_1 (P_i - P_L) \qquad (2\text{-}2)$$

where:

P_i = sediment interstitial phosphorus concentration (M/L^3)

P_s = sediment phosphorus concentration

ε = sediment porosity

K_1 = release rate (L/T)

K_2 = sedimentation rate (T^{-1})

The release of phosphorus from sediment was governed by the concentration gradient between the sediment interstitial phosphorus and the lake water phosphorus. The rate constants K_1 and K_2 were evaluated by in situ tests. A tank was placed in Lake Warner enclosing a portion of the lake water. Phosphorus in the tank was reduced by pumping the lake water within the tank through an activated carbon bed and an anionic exchanger before its return to the tank. After reducing its total phosphorus to a certain level, the exchanger system was discontinued and the tank water mixed by a pump. Measurements of phosphorus vs. time were taken and K_1 was estimated as:

$$K_1 = \frac{\left.\frac{dP_L}{dt}\right|_{\text{initial rate}}}{\frac{\varepsilon A}{V}\,(P_i - P_L)}$$

where V = reactor volume (L^3)

After the phosphorus concentration of the tank water reached steady-state, K_2 was estimated assuming simultaneous sedimentation and release:

$$P_L \text{ steady state} = \frac{\frac{\varepsilon A}{V} K_1 P_i}{\frac{\varepsilon A}{V} K_1 + K_2}$$

A finite difference approach is used to solve the mass balance equations as outlined below. First, equation (2-2),the sediment mass balance, is cast in finite difference form:

$$P_s(t+\Delta t) = P_s(t) + \{ \frac{k_2 P_L(t) V}{D_r A} - \frac{\varepsilon}{D_r} K_1 [P_i(t) - P_L(t)] \}\Delta t \qquad (2\text{-}3)$$

where D_r = reactive depth of sediment (L)

Thus, the conditions for the lake, interstitial, and sediment phosphorus concentrations at time t, with known rates of sedimentation and release, will allow the calculation of the sediment solid-phase phosphorus content at the next time step. The reactive depth incorporated into the equation is an average depth of available phosphorus estimated to be 10 centimeters based on the results of several investigators.

Next, an empirical relationship is used to find the interstitial phosphorus concentration at the new time step. This relationship was found to be linear based on field measurements by Snow and DiGiano (1976), and is written in finite difference form as follows:

$$P_i(t+\Delta t) = \frac{2.10 \times 10^{-3} P_s(t+\Delta t) + 41.87}{\varepsilon} \qquad (2\text{-}4)$$

The sediment phosphorus (P_s) in this equation represents the exchangeable fraction of the total sediment phosphorus.

This fraction was estimated to be 25 percent based on the results of several investigators.

Equation (2-1), *the lake water phosphorus mass balance* may now be used to determine the lake P concentration at the new time step. This is accomplished by assuming that the interstitial phosphorus in the equation remains constant. This assumption allows the first order differential equation to be solved analytically for the lake water phosphorus concentration. This equation is then stated as a finite difference equation using the interstitial sediment phosphorus calculated in equation (2-4) at the new time step:

$$P_L(t+\Delta t) = \frac{P_o + (\frac{\varepsilon A}{V} K_1 P_i(t+\Delta t)\ \bar{T}}{1 + (\frac{\varepsilon A}{V})\ K_1 \bar{T} + K_2 \bar{T}}$$

$$\{1-\exp[\ -\ (\frac{1}{\bar{T}} + \frac{\varepsilon A}{V} K_1 + K_2)\ t]\}$$

$$+\ P_{L_o}\ \exp\ [\ -\ (\frac{1}{\bar{T}} + \frac{\varepsilon A}{V} K_1 + K_2)\ t] \qquad (2\text{-}5)$$

where: P_{L_o} = lake phosphorus concentration at $t = 0$ (M/L^3)

$\bar{T}$ = lake detention time (T)

All variables at time step $t + \Delta t$, the new time step, have now been solved for, and the calculations using equations (2-3), (2-4), and (2-5) are repeated for subsequent time steps. The time step chosen by Snow and DiGiano was ten days, and the model was run for several years.

Snow and DiGiano tested their model to predict the recovery of Lake Warner, after the elimination of the discharge of the Amherst raw wastewater to Lake Warner in 1975. A reduction of lake water phosphorus from 90 μg/ℓ to 50 μg/ℓ was input to the model. The predicted time for this reduction in phosphorus concentration to occur was fourteen years.

CHAPTER III

BACKGROUND AND METHODOLOGY

Lake Warner, located in Hadley, Massachusetts, is shown in Figure 2. It is about one and one-half miles long and varies from one hundred to several hundred feet in width. It lies between Mt. Warner and the village of North Hadley. The lake was actually formed by a dam built on the Mill River in the early 1800's. Less than one-half mile downstream from the dam, the Mill River empties into the Connecticut River. The lake is shallow, ranging in depth from 3 to 10 feet, with the deepest portion being the old river bed.

The basin drained by the Mill River and Lake Warner has an area of about 32 square miles in the towns of Amherst, Leverett, Hadley, and Sunderland, including the University of Massachusetts. The land usages as shown in Table 1 were estimated by Jubinville (1973).

Table 1

Land Usage of Lake Warner Drainage Basin

Land Use	Percent of Total Area
Non Agricultural forest	50
Agricultural (crops)	20
Agricultural (dairy farms)	20
Urban	10

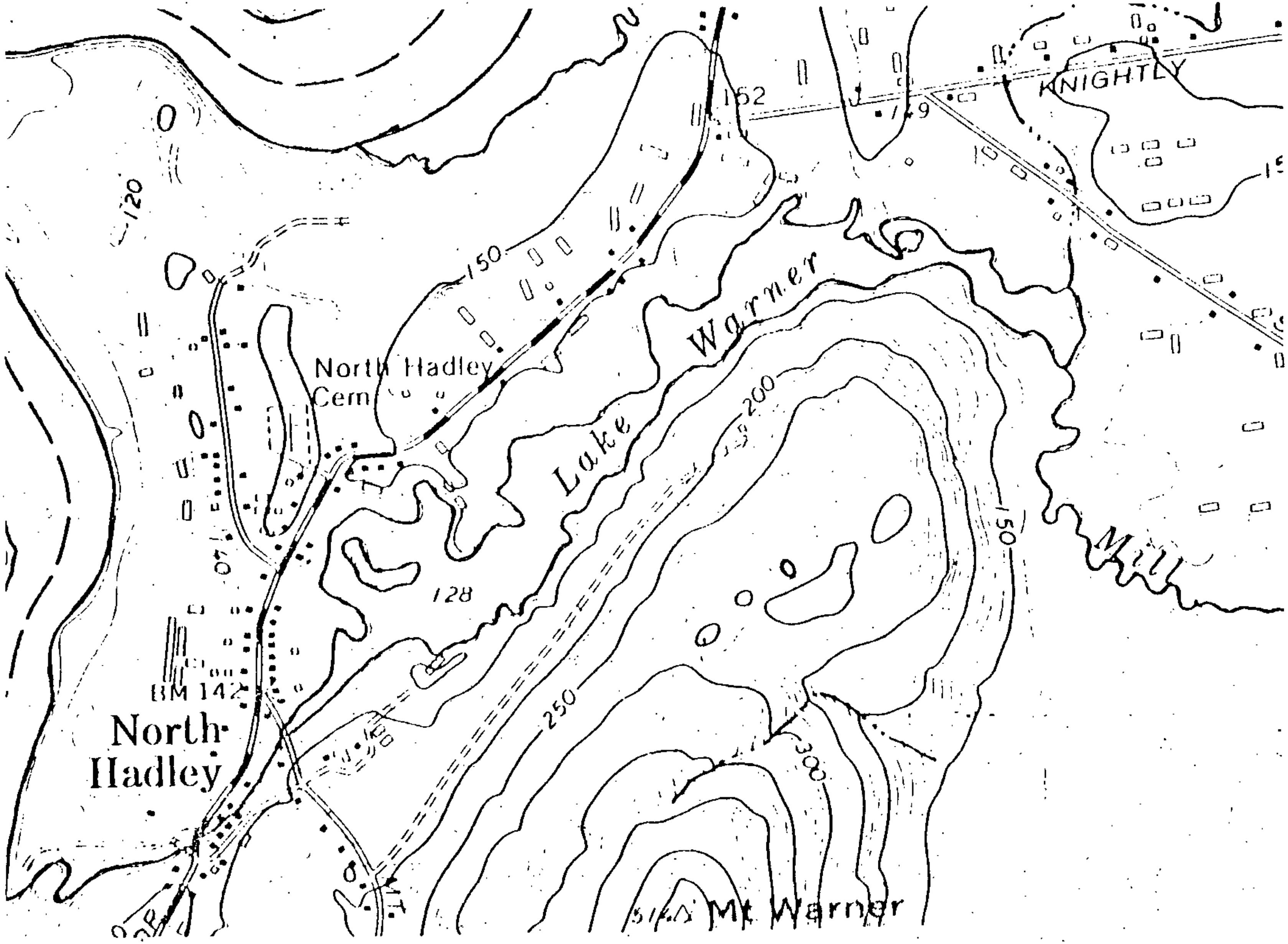

Figure 2. Location Map of Lake Warner

Lake Warner is closely set between Mt. Warner and the Connecticut River, and only a small percentage of the entire 32 square mile drainage basin drains directly to the lake. Most of the basin drains into the Mill River. The river is thus the only major contributor of phosphorus to Lake Warner. Natural runoff, particularly over agricultural land, and the raw wastewater bypassing the Amherst Wastewater Treatment Plant were the major sources of phosphorus entering Mill River. The bypassing of the Amherst wastewater roughly doubled the normal Mill River phosphorus content as estimated by Snow and DiGiano (1976). The same authors observed algal blooms in Lake Warner throughout the summer months in 1974, indicating that the lake was highly eutrophic.

Field Measurement Program

The Amherst Wastewater Treatment Plant stopped its wastewater bypassing in November 1975. This reduction in phosphorus loading should allow the lake to begin its recovery. A field measurement program was designed to monitor the recovery of Lake Warner. The sampling and analysis focused on inflow and outflow of phosphorus to and from the lake, as well as the phosphorus contents of the lake water and sediments. Other water quality parameters were also measured to monitor the general quality of the lake water. The Mill River water at the inlet and outlet of Lake Warner was sampled weekly from October 1976 to September 1977, except in June and July when it was sampled daily. The location of the sampling stations, No. I (inlet) and No. O (outlet) is shown in Figure 3. Total and orthophosphate phosphorus were analyzed for all the samples, while pH, alkalinity, DO, turbidity, ammonia-N, nitrate-N, temperature, and specific conductance were measured for a number of selected samples. The inflow and outflow rates of Lake Warner were also measured weekly, from December, 1976, to September, 1977, to coincide with the phosphorus measurements. These flow measurements were also increased to daily during June and July of 1977. The results of measurements are presented in Table 5 of Appendix A. These data will be analyzed, graphically displayed and discussed in detail later in this report.

Lake Warner was sampled at four stations in the Lake from October, 1976 to September, 1977. The location of the four sampling stations, Nos. 1, 2, 3, and 4 is shown in Figure 3. The sampling was weekly during June and July, 1977, and monthly for the rest of the sampling period. Emphasis was placed on measurement of total and orthophosphate phosphorus of both lake water, and the interstitial water of lake sediments. Other parameters measured were DO, temperature, Secchi-disc depth, pH, alkalinity, turbidity, ammonia-N, nitrate-N, and specific conductance of lake water, and total phosphorus of lake sediment. The results

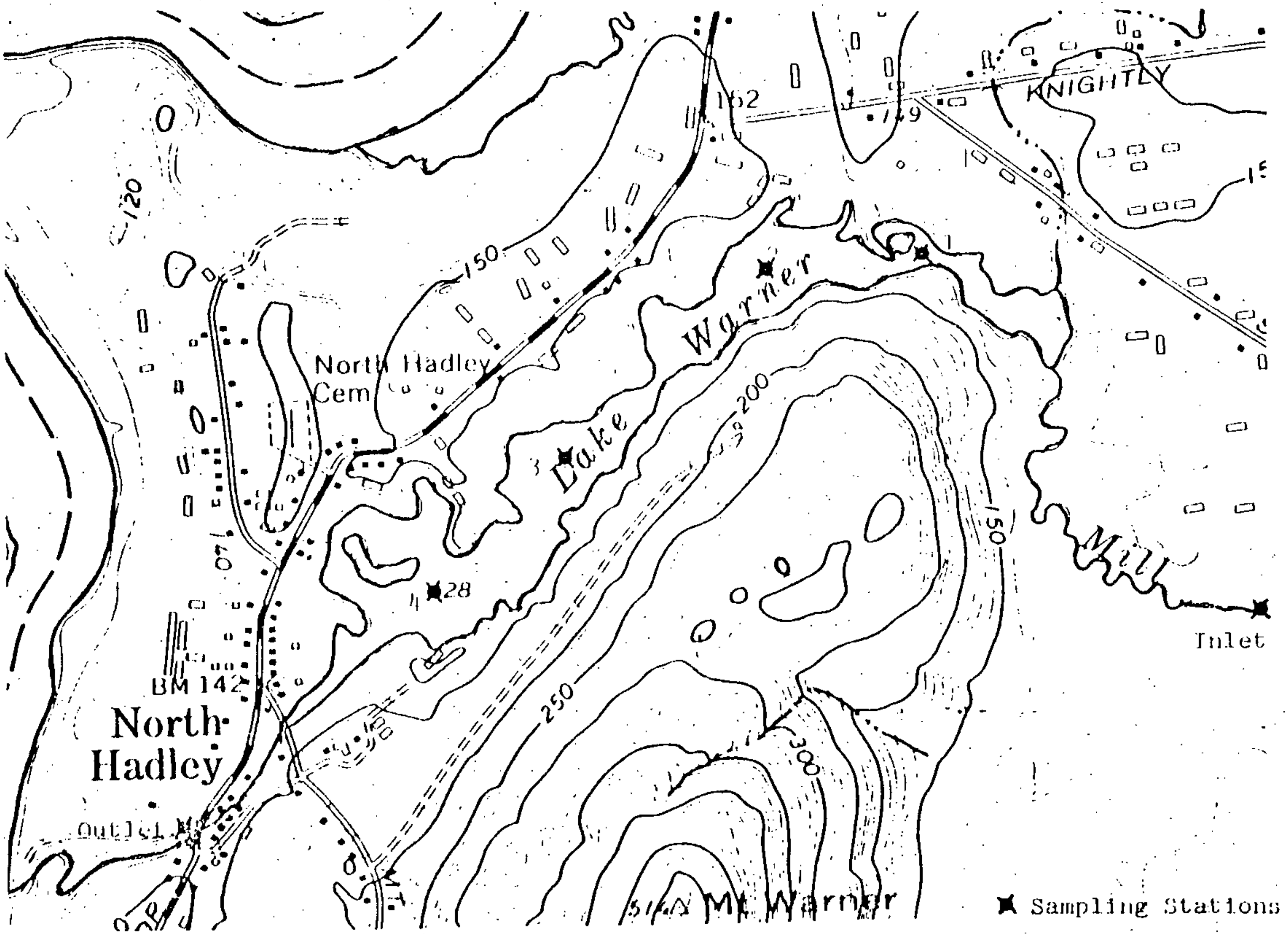

Figure 3. Location of Sampling Stations

of Lake Warner measurements are presented in Table 6 of Appendix A. These data will also be analyzed, graphically displayed and discussed in detail later in this report.

Sampling and Analysis Methodology

Stream gaging was done at the lake inlet and outlet using the USGS mid-section method (Buchanan et al., 1969). The Mill River cross sections at the inlet and outlet of Lake Warner were divided into twelve 3-foot sub-sections. A Pygmy current meter was used for velocity measurement at six-tenths of the water depth of each sub-section. Permanent benchmarks were established and a staff gage was calibrated versus flow rates to establish stream-discharge rating curves. Results are presented in Appendix B.

Total orthophosphate and phosphorus were measured spectrophotometrically, using the EPA Single Reagent Method (1971). Since the most measured phosphorus concentrations were greater than 20 μg/ℓ, the more sensitive isobutanol extraction method was not necessary. Sediment interstitial water was separated by centrifuging sediment samples for 30 minutes at 10,000 rpm. Sediment samples were ashed before digestion for total phosphorus analysis. Ammonia analysis was done by direct Nesslerization. Of the other tests, samples were brought to the laboratory for alkalinity, pH, and turbidity determinations, while dissolved oxygen, temperature, and Secchi-disc depth were measured *in situ*.

CHAPTER IV

ANALYSIS OF DATA

Hydraulic and water quality data collected are presented in this chapter. The data include Mill River flow, Mill River water quality, Lake Warner water quality, and Lake Warner sediment quality.

Stream Gaging

Mill River flow was measured at the inlet and outlet of Lake Warner. These flows and their respective measured phosphorus concentrations are needed to calculate the mass balance of phosphorus in Lake Warner.

The stage-discharge curves (stream-discharge rating curves) of Mill River at the inlet and outlet of Lake Warner are presented in Figures 4 and 5, which are to be used for river-flow measurements. The field measurements are presented in Appendix B. The hydraulics involved can be explained by Manning's equation which describes open channel flow as follows:

$$Q = \frac{1.49}{n} A_c R^{2/3} S^{1/2} \qquad (4\text{-}1)$$

where:

Q = flow (cu. ft./sec.)

n = Manning's "n' coefficient

A_c = cross-sectional area (sq. ft.)

R = hydraulic radius (ft.)

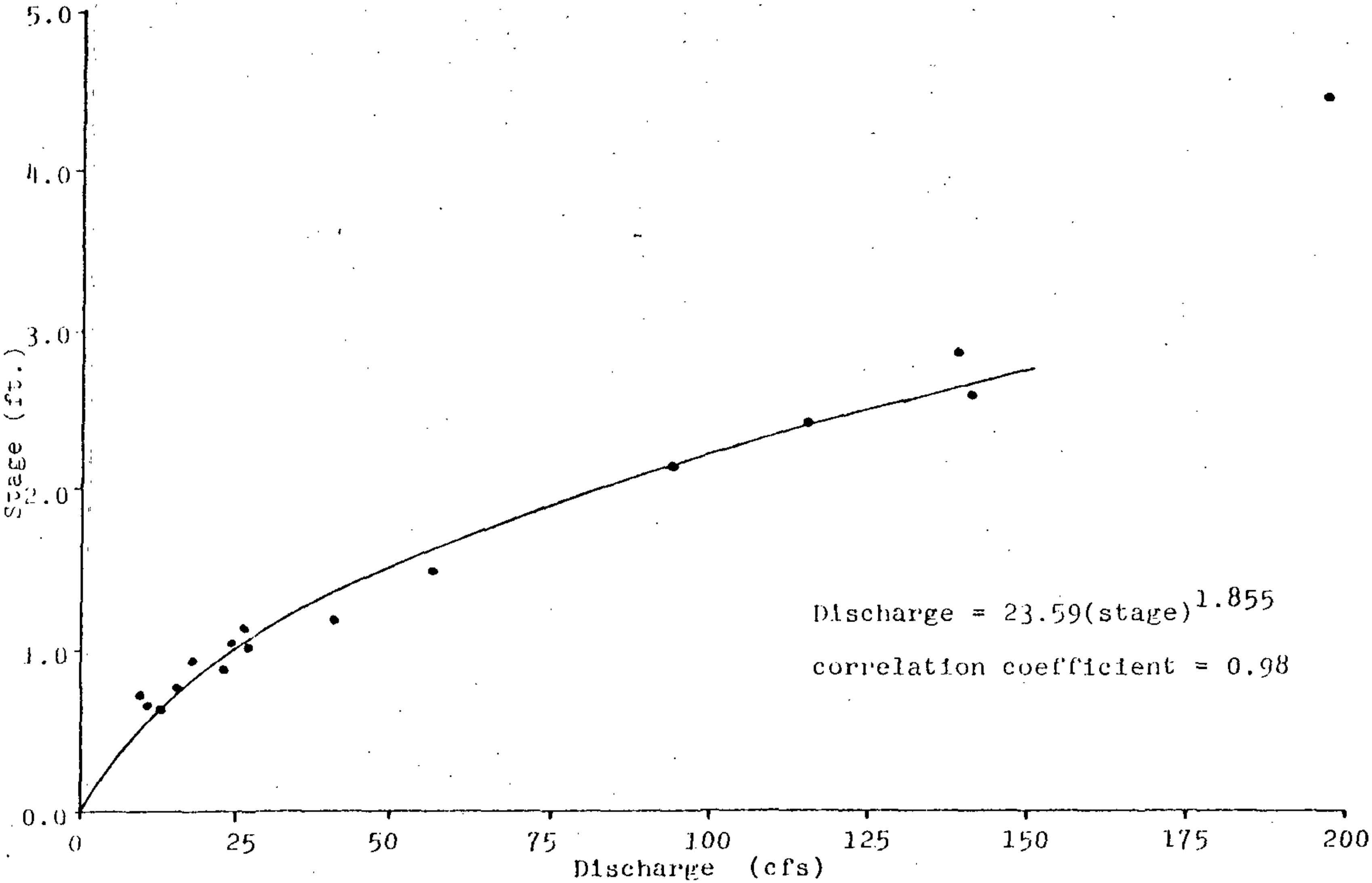

Figure 4. Stage-Discharge Curve of Mill River at the Inlet of Lake Warner

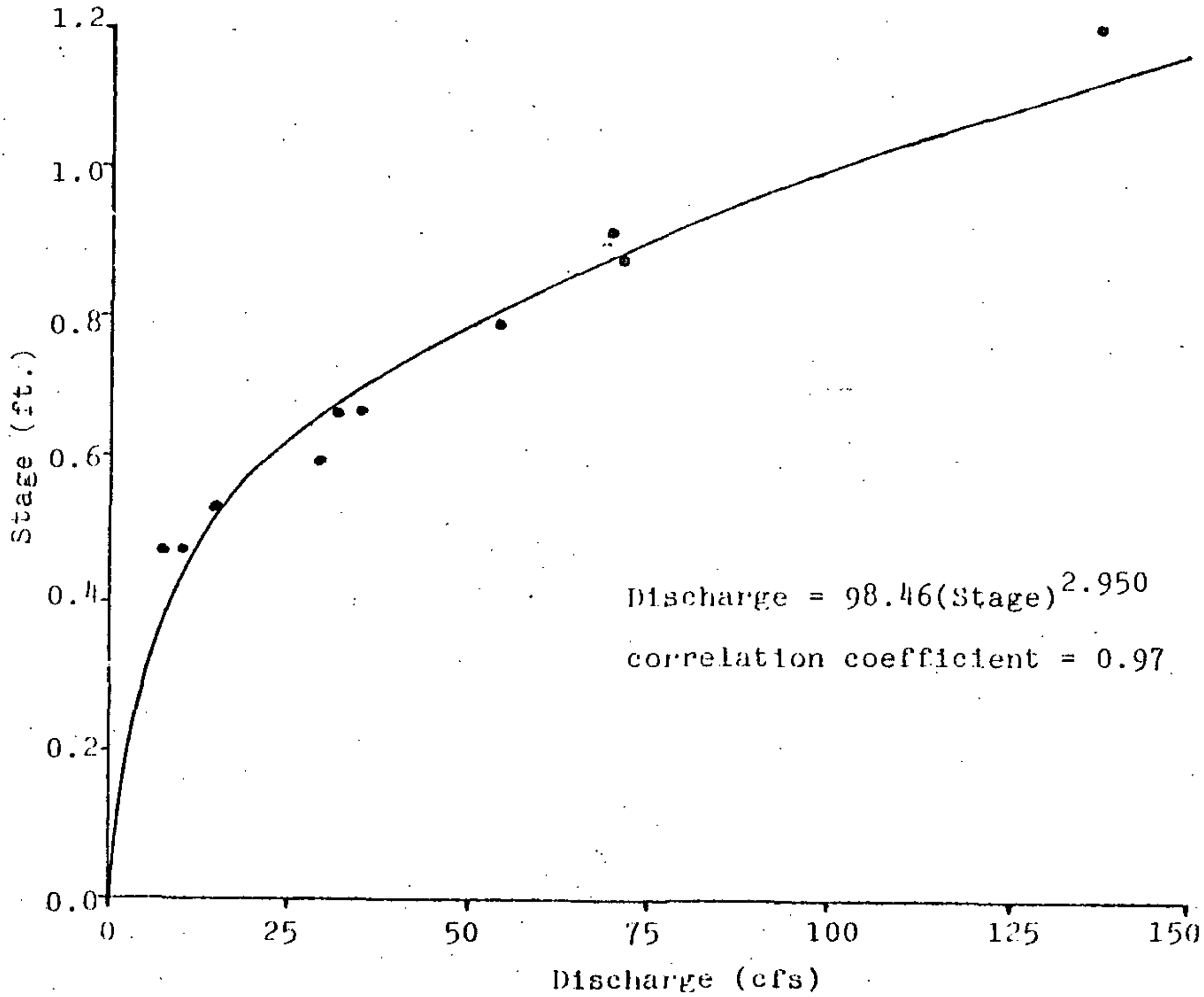

Figure 5. Stage-Discharge Curve of Mill River at the Outlet of Lake Warner

S = channel slope (ft./ft.)

For retangular channels, the hydraulic radius and area are:

$$R = \frac{Bh}{2h+B} \tag{4-2}$$

and

$$A_c = Bh \tag{4-3}$$

where:

h = channel depth (ft.)

B = channel width (ft.)

Substituting Equations (4-2) and (4-3) into Equation (4-1), Manning's equation becomes:

$$Q = \frac{1.49}{n} (Bh) \left(\frac{Bh}{2h+B} \right)^{2/3} S^{1/2} \tag{4-4}$$

If B, n, and S are constant, Equation (4-4) becomes:

$$Q = ah^c$$

where a and c are coefficients. This relationship between Q and h should be a straight line if plotted on logarithmic scales.

The data expressed in the relationship of "$Q = ah^c$" were statistically analyzed by the method of least squares. At the inlet station, sixteen data points exhibited a correlation coefficient of 0.978. At the outlet gaging site, ten points resulted in a correlation coefficient of 0.968. In both cases, the correlation coefficients showed that the river flows can be estimated to a reasonable degree of accuracy by measuring the stage heights of the calibrated staff gages at the inlet and outlet stations of Lake Warner. The flow measurements for establishing the stream-discharge rating curves are summarized in Appendix B, and all measured and estimated flows are included in Appendix A.

Mill River Water Quality

The measured data on the water quality of Mill River are summarized in Appendix A. Ammonia and nitrate-nitrogen concentrations were very high, roughly twenty or more times that of phosphorus, supporting the view that phosphorus was the limiting nutrient in Lake Warner.

The influent and effluent total and orthophosphate phosphorus are plotted vs. time (weekly) and rainfall (over 0.5 inches) from October 1976 through May 1977 in Figures 6 and 7. Immediately after rainstorms, large increases in phosphorus concentration occurred, apparently due to washout by intensive runoff. During this study, the average influent total phosphorus concentration was about 50 μg/ℓ, a significant drop as compared to the influent total of 90 μg/ℓ measured by Snow and DiGiano in 1974. The drop was probably a result of the discontinuation of wastewater-bypassing to the Mill River from the Amherst Wastewater Treatment plant. Total phosphorus concentration of 50 μg/ℓ with about 50 percent in the form of orthophosphate is the lowest to be expected under natural conditions.

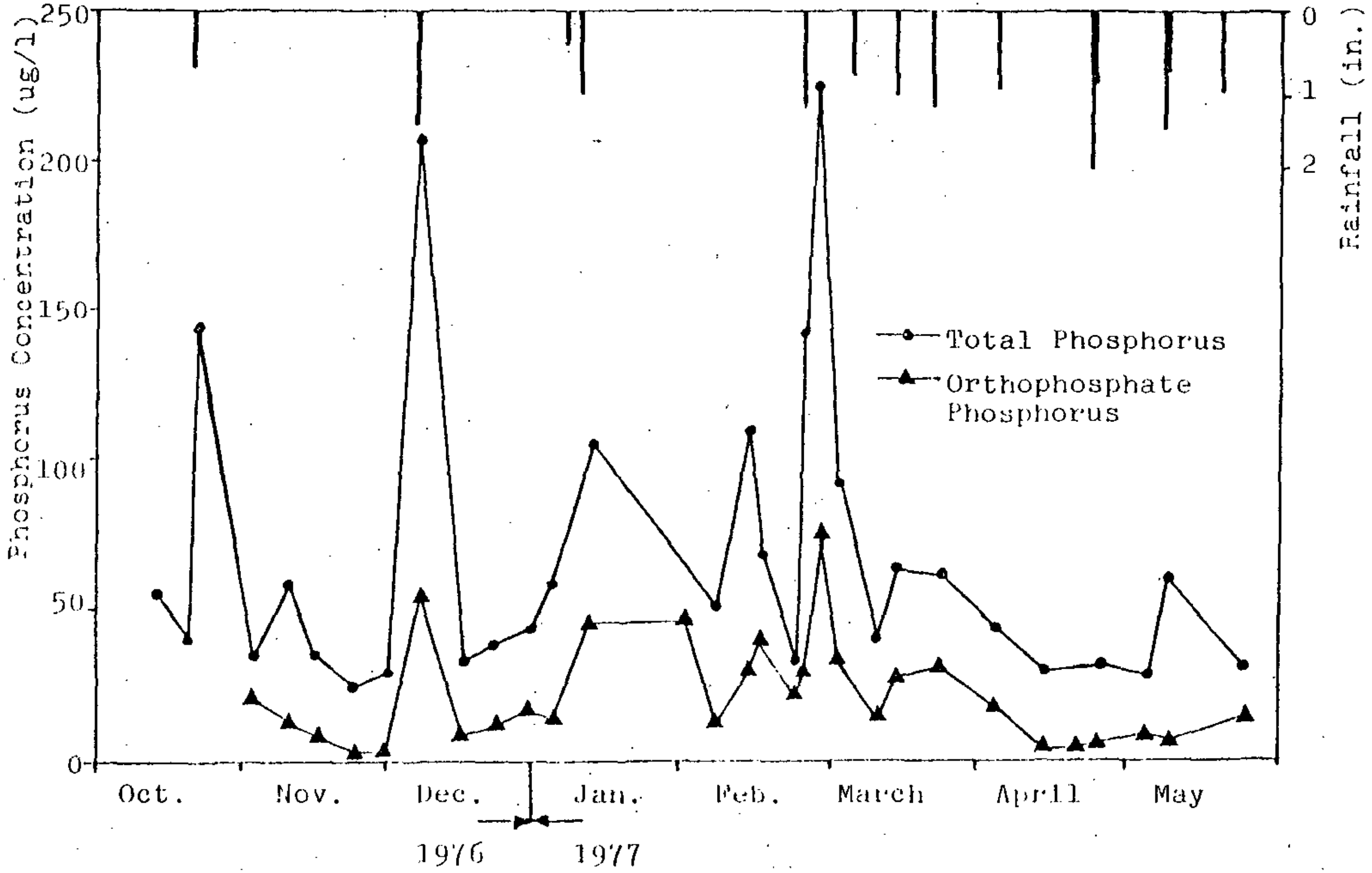

Figure 6. Influent Total and Orthophosphate Phosphorus

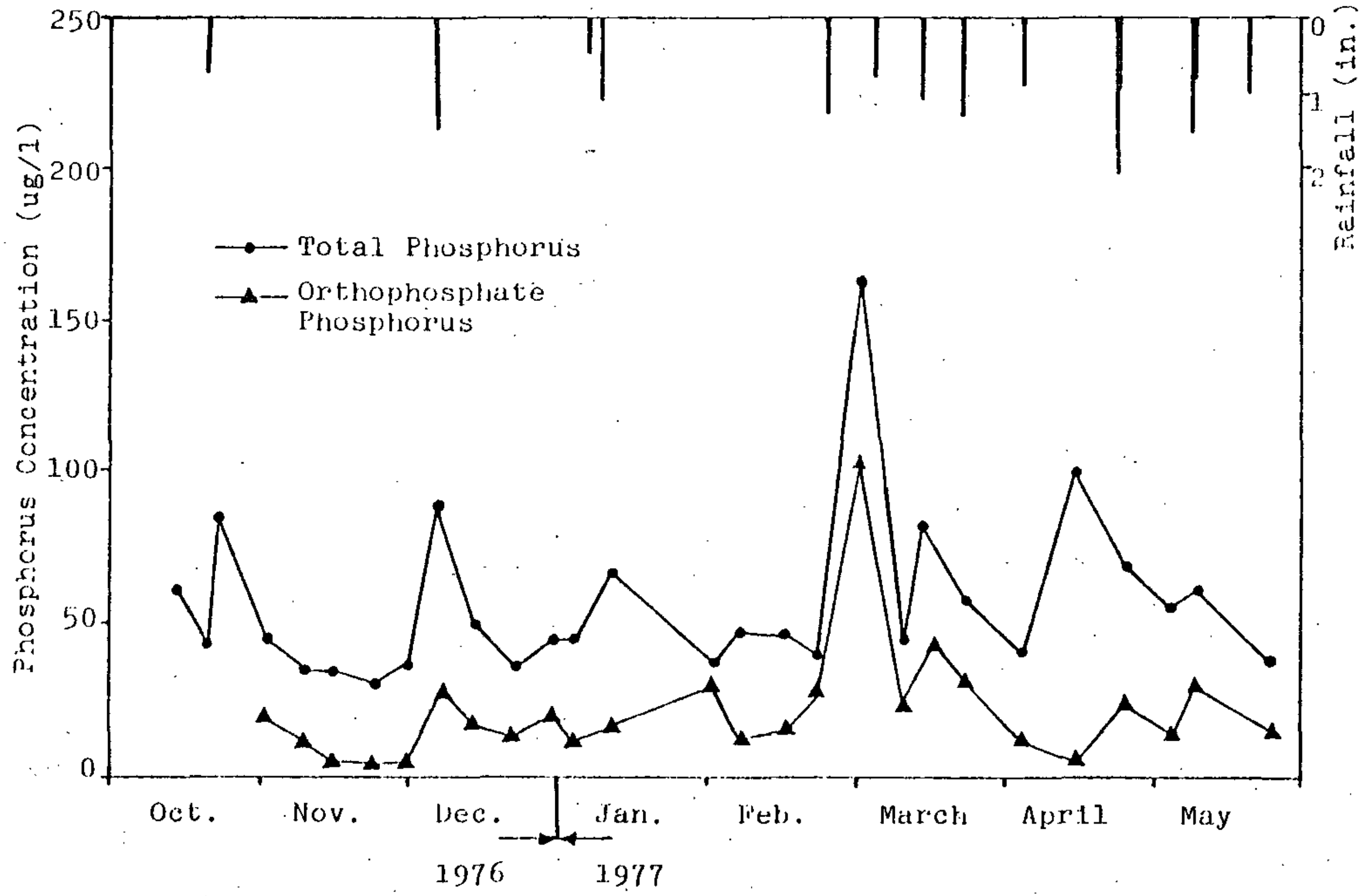

Figure 7. Effluent Total and Orthophosphate Phosphorus

The "dampening" effect that the lake had on phosphorus concentrations can be observed by comparing the influent plot (Figure 6) and the effluent plot (Figure 7). The influent concentrations show sharp peaks and valleys, while the effluent variations are lessened.

The influent and effluent total phosphorus concentrations and flow rates as well as the rainfall measured during the intensive summer sampling period are plotted vs. time in Figures 8 and 9. In Figure 8, it is shown that increases in influent phosphorus concentration occurred during rainfalls but before the arrival of high flows in late June and near the middle of July. Due to the high Mill River flows, the total amount of phosphorus which entered Lake Warner was substantial during this period.

The effluent phosphorus concentration remained fairly constant at about 50 μg/ℓ. The sharp influent phosphorus increases were dampened by the lake. There was an increase during the high flow period in mid-July. This was probably due to the flushing of deposited phosphorus under the high flow condition, and the short detention time of about two days.

The Mill River was additionally sampled during a rainstorm and a dry period respectively to show their effects on phosphorus concentration of the Mill River water.

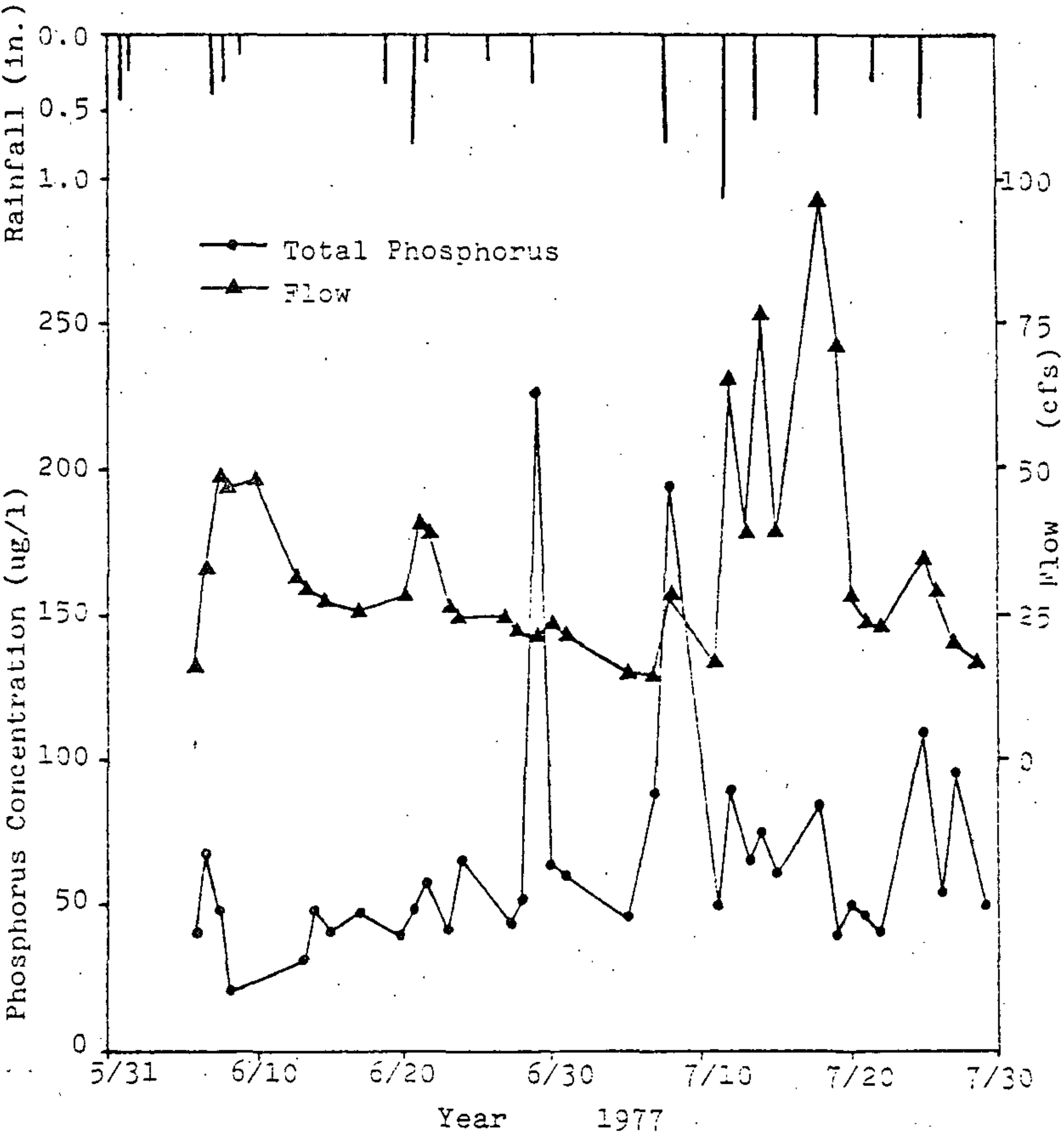

Figure 8. Influent Total Phosphorus and Flow

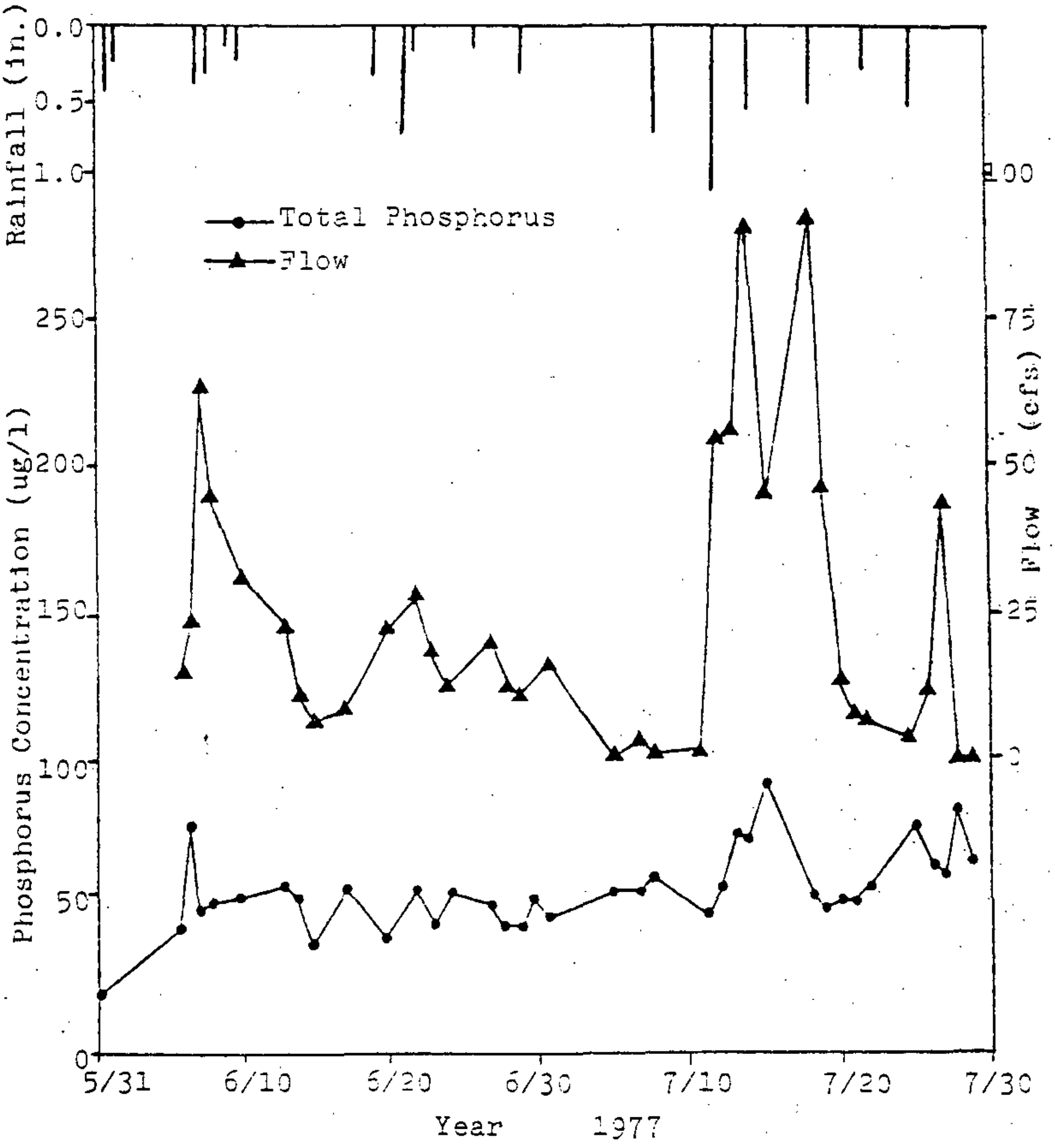

Figure 9. Effluent Total Phosphorus and Flow

As shown in Figure 10, the phosphorus concentration increased in the direction of the flow of Mill River through Lake Warner during dry weather and it had a surge of phosphorus increase during wet weather. Thus, even without the discharge of wastewater, the natural runoff can still carry a significant amount of phosphorus to Lake Warner.

Lake Warner Water Quality

Lake Warner water quality data are presented in Appendix A. The water quality was generally good with dissolved oxygen near saturation at both surface and bottom throughout the year. In July and August, evidence of algal photosynthesis was detected as the lake water was supersaturated with DO, although visible algal blooms were only present in mid-July for less than one week. Before the Amherst wastewater-bypassing had ceased, blooms were evident through much of the summer and bottom water of the lake was anaerobic (Snow and DiGiano, 1976).

Total and orthophosphate phosphorus concentrations were measured monthly from October 1976 through September 1977, and more frequently during June and July, and the results are presented in Figure 11. The plotting shows both the average and the range of phosphorus concentrations at the four sampling stations (see Figure 3). In the period from October 1976 to June 1977, the total and orthophosphate phosphorus remained fairly constant. In June,

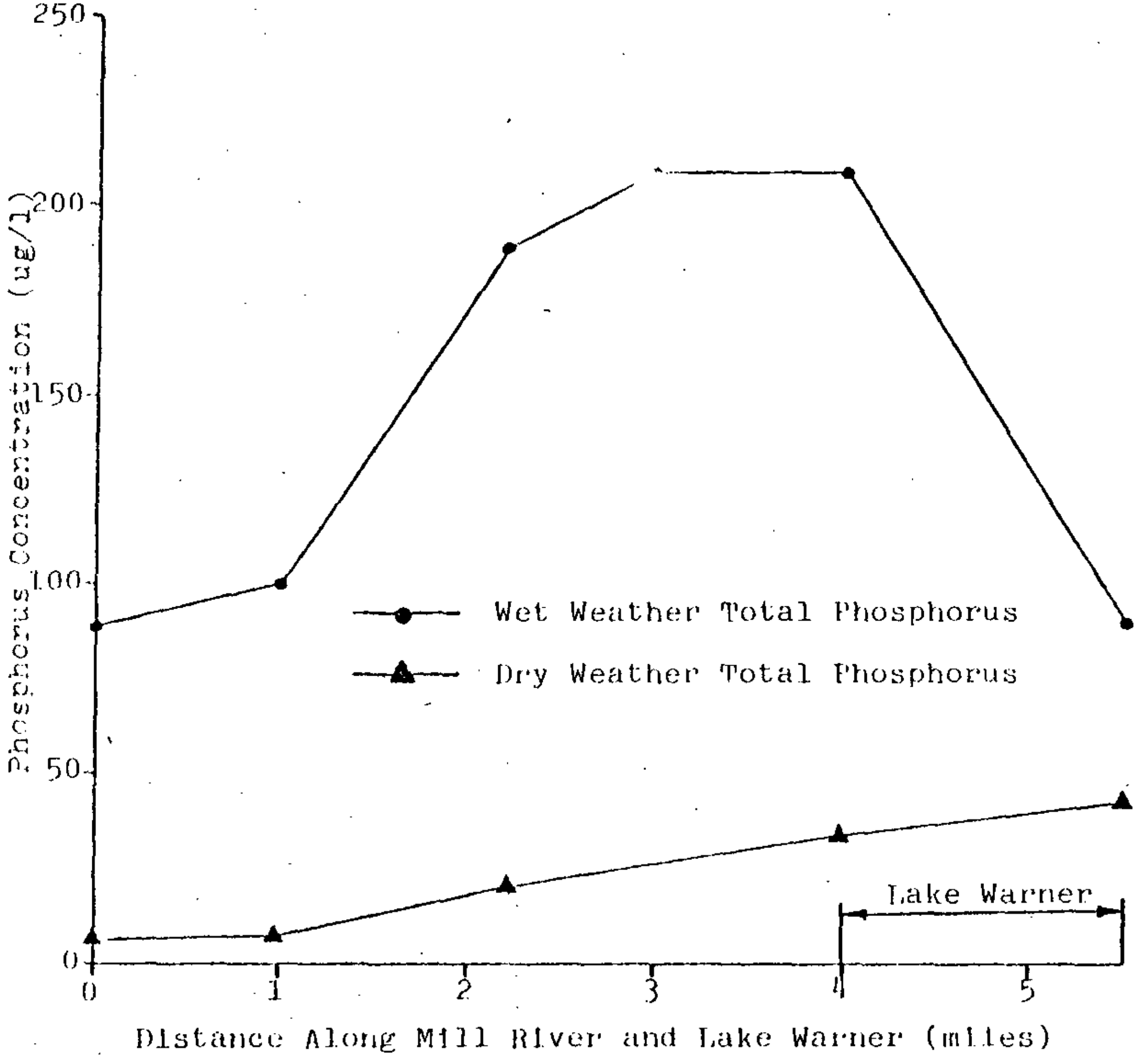

Figure 10. Effect of Rainfall on Mill River Phosphorus Concentration

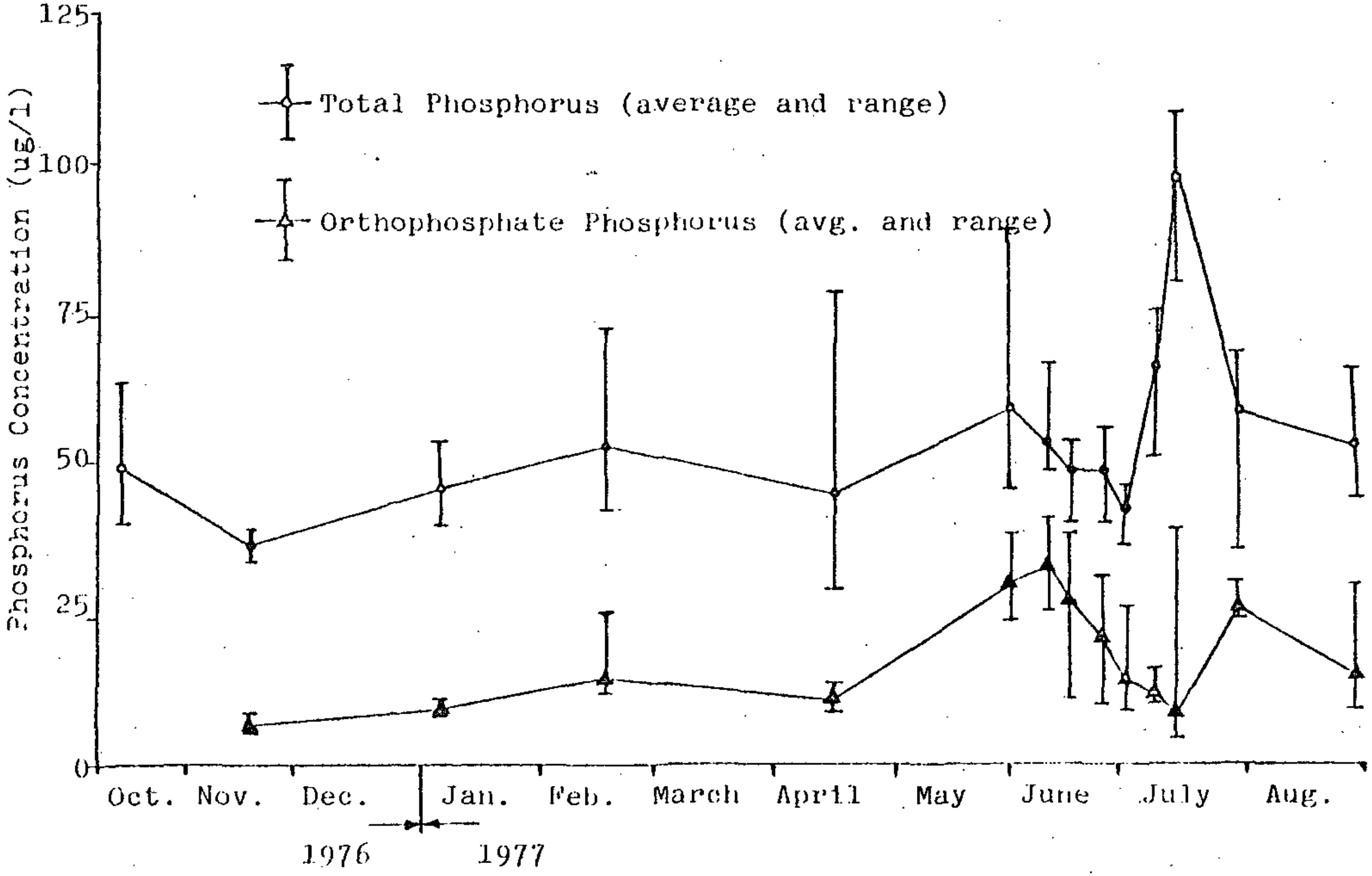

Figure 11. Lake Warner Total and Orthophosphate Phosphorus

there was a general decrease, probably due to the decrease of Mill River flow. In early July, there was a dramatic rise in total-P and a drop in ortho-P, during which period an algal bloom was observed in the lake, and evidently algal uptake of reactive P had caused such a shift of phosphorus. Subsequent die-off and settling of the algae occurred later in the month, resulting in a lower total-P and the recovery of ortho-P to its normal percentage of total P. The average total phosphorus concentration was about 50 μg/ℓ, which was significantly lower than the average value of 90 μg/ℓ measured by Snow and DiGiano in 1974.

In general, concentrations of phosphorus were about the same in all parts of the lake. The phosphorus data under some specific conditions are plotted in Figure 12, showing the phosphorus concentrations during an algal bloom in mid-July and after a heavy rainstorm in September. During the algal bloom, there was a wide difference between total phosphorus and orthophosphate phosphorus due to algal uptake of reactive phosphorus. It should be noted that the samples taken during the bloom were not filtered before phosphorus measurements, and therefore both particulate and dissolved phosphorus were included in the tests. During the rainstorm, both total and orthophosphate values were high, and the values increased towards the outlet of the lake. This was probably due to the fact that samples were taken after the

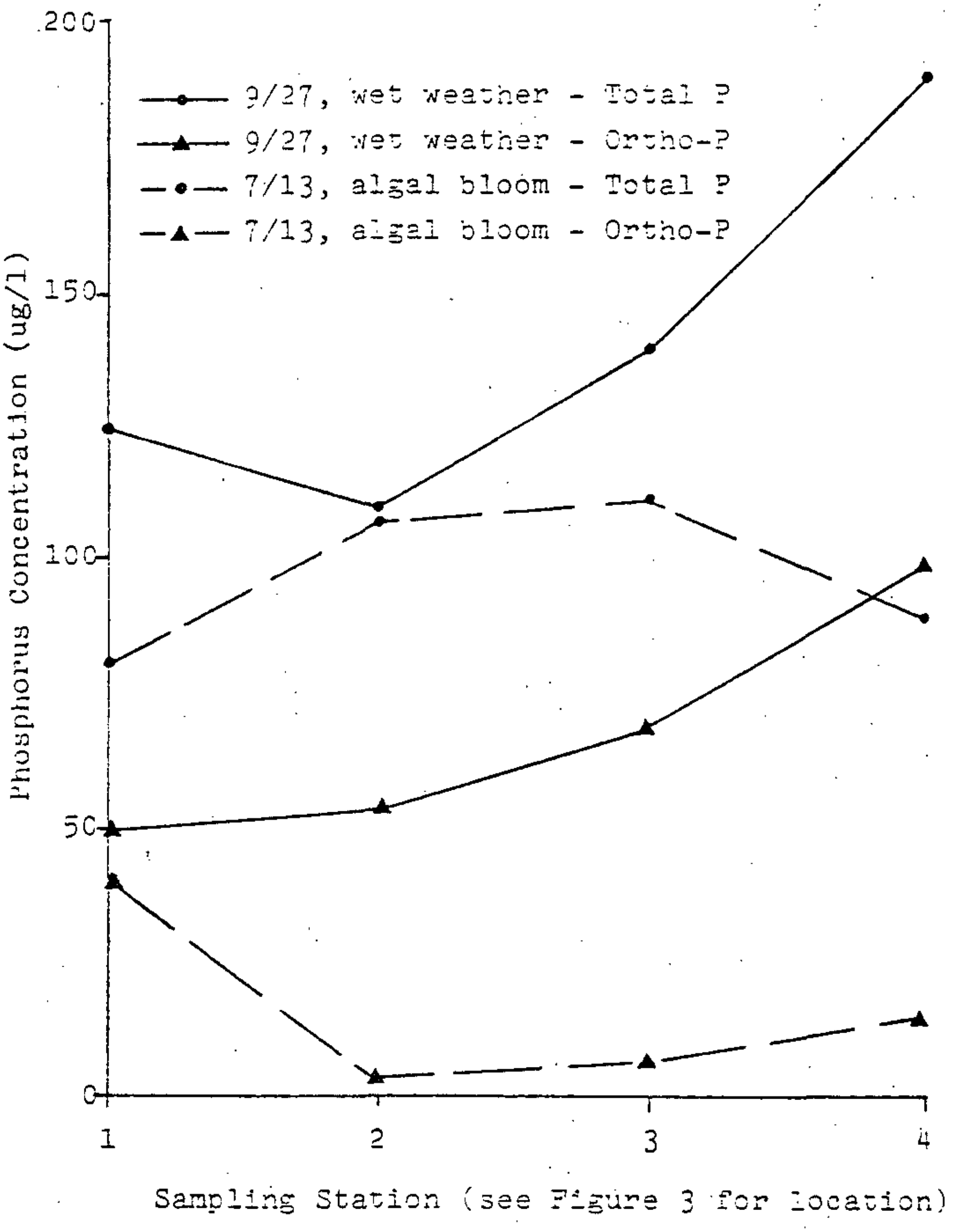

Figure 12. Total and Orthophosphate Phosphorus in Lake Warner Under Specific Conditions

rainfall, and influent phosphorus concentration was becoming lower while the high phosphorus-concentration water was still traveling toward the outlet.

Lake Warner Sediment Quality

Lake sediment samples were collected at the four sampling stations during the summer of 1977. Total and orthophosphate phosphorus in the interstitial water of the sediment samples were measured. The highly variable phosphorus contents of the interstitial water of the sediments sampled at different locations in the lake are shown in Figure 13. However, the pattern of variation was not discernable. Most of the data fall in the range of 150-300 μg/ℓ for total phosphorus and 50-150 μg/ℓ for orthophosphate phosphorus. Generally the orthophosphate phosphorus increased with total phosphorus.

Total phosphorus of the sediment interstitial water is plotted against its orthophosphate component in Figure 14. Also plotted in this figure is the correlation line of Lake Warner sediment phosphorus measured by Snow and DiGiano (1976). It is obvious that nearly all data points measured in this study fall below the Snow and DiGiano data. A significant reduction of the phosphorus concentration in the interstitial water of Lake Warner sediments has occurred since 1974.

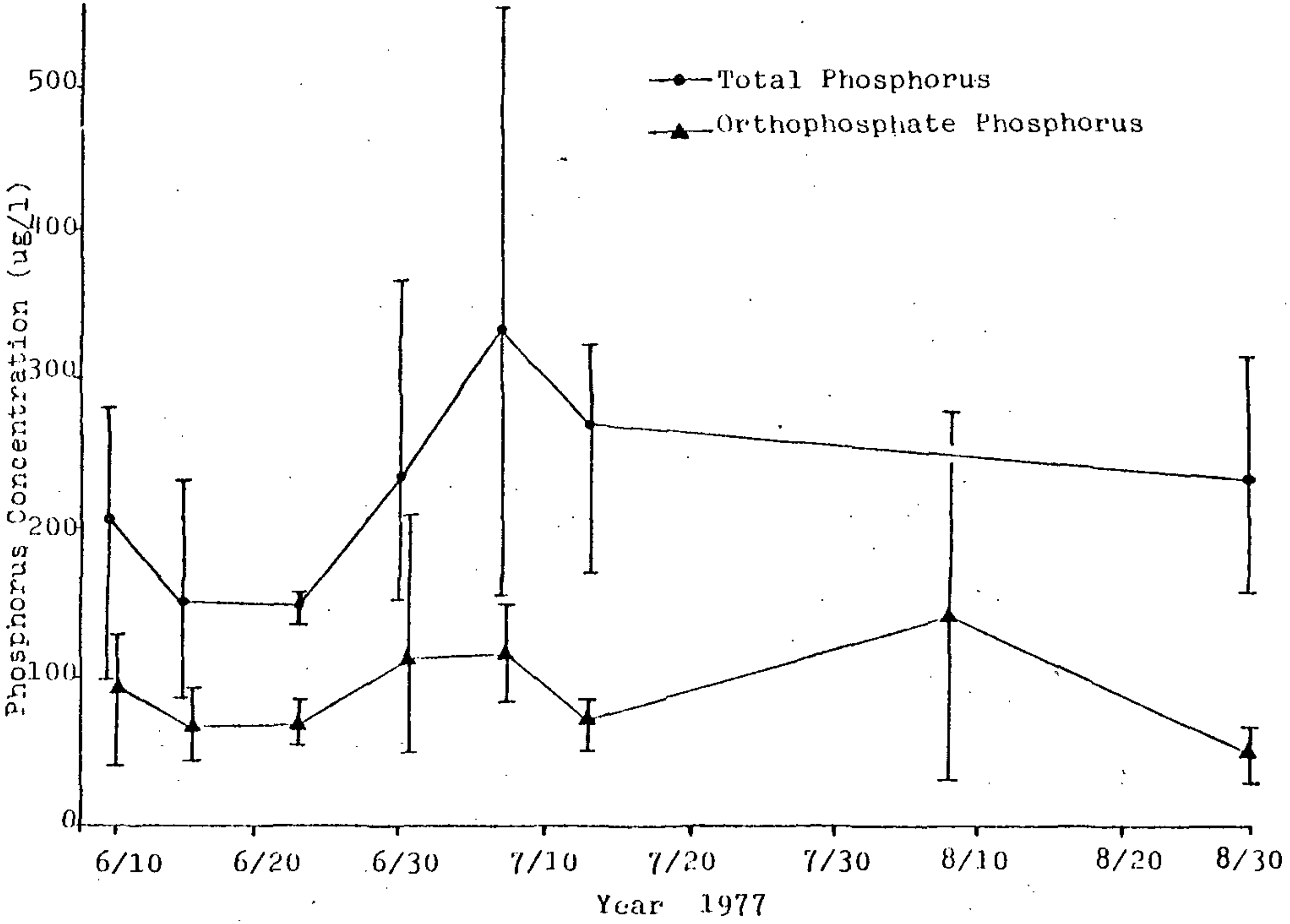

Figure 13. Total and Orthophosphate Phosphorus in Interstitial Water of Lake Warner Sediments

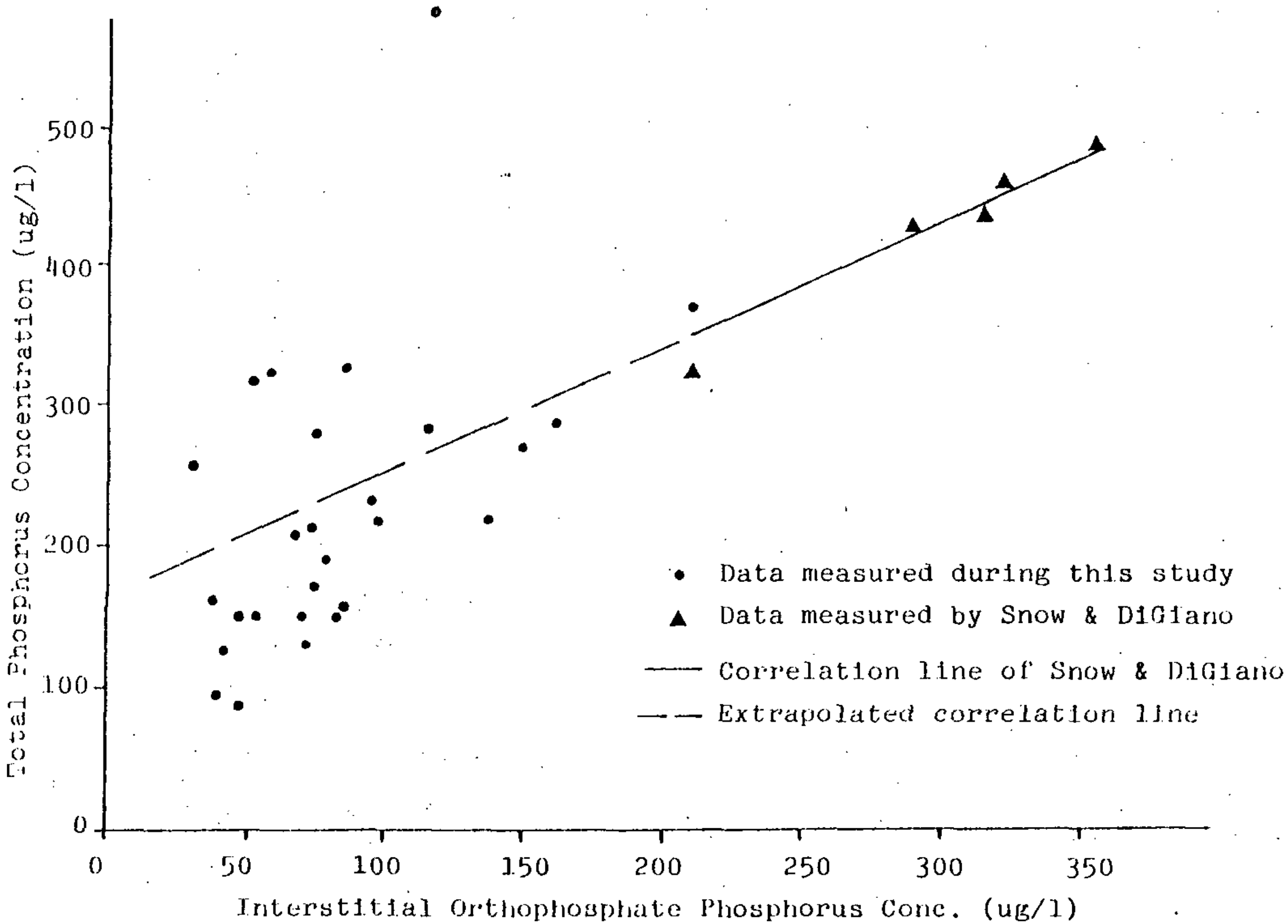

Figure 14. Total vs. Orthophosphate Phosphorus in Interstitial Water of Lake Warner Sediments

The total phosphorus contents of dried lake sediments sampled at the four sampling stations were measured. Most samples were analyzed in duplicate (two different portions of a sediment sample). Results are listed in Table 2. Also shown are the total and orthophosphate phosphorus of the interstitial water of the same samples. The total phosphorus of dried sediments is plotted against the total and orthophosphate phosphorus of the sediment interstitial water in Figure 15. Correlation lines are estimated, and as shown, the correlation is poor. Also shown in Figure 15 is the correlation given by Snow and DiGiano (1976) on the basis of their data taken in June, 1974.

It is surmised that the shift in phosphorus equilibrium between the sediment and its interstitial water of Lake Warner from 1974 conditions to present conditions has resulted from the decrease in phosphorus loading to the lake after the cessation of the Amherst wastewater discharge to the Mill River. Dissolved oxygen measurements taken at the surface and bottom of the lake throughout this study showed a practically uniform DO level near saturation, even during the summer months and in the deepest portions of the lake. It was a significant improvement over the values measured earlier by Snow and DiGiano (1976), which showed anoxic conditions existing at the lake bottom. Also as shown in Figure 15, the phosphorus levels of dried sediments were much higher and of interstitial waters much

Table 2

Sediment Phosphorus Data

	Total P of Dried Sediments (mg/g)					P of Interstitial Water	
Date	Station	Test 1	Test 2	Test 3	Avg.	Total P (μg/ℓ)	Ortho-P (μg/ℓ)
6/10/77	1	3.70	2.69	2.92	3.10	280	116
	2	1.01	1.02		1.02	218	128
	3	2.47	2.18	1.87	2.17	214	72
	4	1.29	2.05		1.67	98	40
6/15/77	1	2.37	2.19		2.28	234	95
	2	2.04	2.40		2.22	83	45
	3	2.20	2.66		2.43	153	71
	4	0.88			0.88	125	42

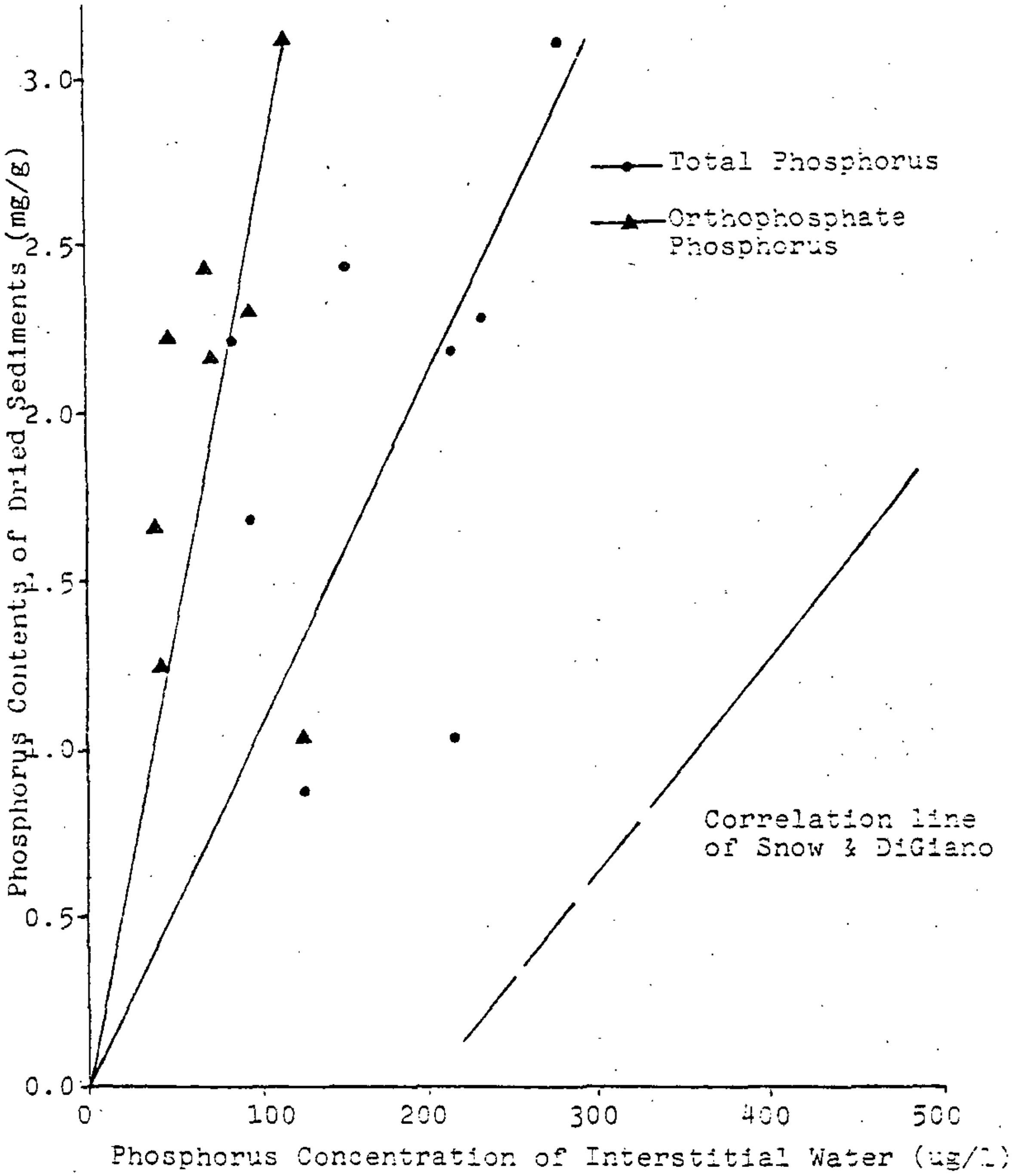

Figure 15. Phosphorus Content of Dried Sediments vs. Phosphorus Concentration of Sediment Interstitial Water

lower than their counterparts as presented by Snow and DiGiano.

Apparently, the oxic condition which now exists throughout the lake has increased the phosphorus adsorption capacity of the sediment and inhibited its ability to release phosphorus into the lake water. In the work by Lung, Canale, and Freedman (1977), it was indeed found that the kinetic coefficient which described the change of particulate sediment phosphorus to the dissolved form in the interstitial water varied inversely with the DO content of the hypolimnion. This reaction slowed to zero in the winter months and reached its peak in the summer months when the hypolimnion DO was near zero.

Ku and Feng (1975) have shown that Lake Warner sediment was highly sensitive to the redox potential. A low redox potential enhances the release of phosphorus from sediment to interstitial water and then to lake water. In 1974, Snow and DiGiano (1976) reported negative redox potential of Lake Warner sediments. During this study a series of measurements taken on May 31, 1977 showed that the redox potential had raised to positive values varying from about 0 to 120 mv.

Summary

The data collected in this study have shown the average phosphorus concentration of Lake Warner water was reduced to about 50 μg/ℓ from the value of 90 μg/ℓ. measured by Snow and DiGiano in 1974. This significant decrease is mainly due to the elimination of the Amherst wastewater discharge to the Mill River in November 1975. Increased dissolved oxygen in lake water and raised oxidation-reduction potential of lake sediments are responsible for the reduction of phosphorus release from the sediment solids to overlying water via the sediment interstitial water and result in the accumulation of phosphorus in the sediment solids.

The average phosphorus concentration of 50 μg/ℓ in Mill River is about as low as can be expected under the present conditions. This concentration may not be high enough to sustain algal blooms, but a condition of low dissolved oxygen in the lake would increase the phosphorus release from the sediment and any cause to lower the dissolved oxygen could trigger algal blooms by the phosphorus released from the sediments in addition to natural introduction of nutrients. The algal blooms observed in Lake Warner in July 1977 may have been the result of low dissolved oxygen due to high temperatures and low flow.

The data collected in this study are to be used to modify the role of the sediments in the phosphorus modeling of Lake Warner by Snow and DiGiano in 1974.

Phosphorus mass balance. The basic equation for mass balance of a chemical constituent can be written as follows:

$$\begin{matrix}\text{Rate of} \\ \text{Mass Input}\end{matrix} - \begin{matrix}\text{Rate of} \\ \text{Mass Output}\end{matrix} \pm \begin{matrix}\text{Rates of} \\ \text{Reaction}\end{matrix} = \begin{matrix}\text{Rate of} \\ \text{Accumulation}\end{matrix} \qquad (4\text{-}5)$$

All terms in this equation can be calculated using measured data except for the reaction rates. The rates of phosphorus input and output to the lake can be calculated by multiplying measured phosphorus concentrations at the lake inlet and outlet by respective measured flow rates. The mass accumulation rate can be calculated from the measured change in lake water phosphorus concentration over time. On the basis of the above mass balance equation, a net phosphorus reaction rate can be obtained. A positive reaction rate indicates a net release of phosphorus from the sediment to the lake water. A net negative reaction rate indicates net settling of phosphorus to the sediment.

A mass balance analysis will be performed using the data collected during the intensive sampling period of June and July 1977. The summer time is a critical period for algal growth, since high temperature, lower dissolved oxygen, and higher pH due to algal activity all tend to favor the release of sediment phosphorus. The data collected in weekly samplings are not adequate for mass balance analysis because each sample was individual due to the long weekly intervals.

The variation of influent and effluent phosphorus mass flow rates of Lake Warner are shown in Figure 16. A phosphorus mass flow rate is the product of the measured flow and its phosphorus concentration. The very low effluent phosphorus flow rate in early July 1977 is a reflection of the low flow conditions. Much of the inflow during that period was stored in the lake or lost from the system, perhaps due to evaporation or contribution to groundwater. This situation may have touched off the algal bloom which was observed shortly thereafter.

For several time periods, the rate of phosphorus mass input, rate of phosphorus mass output, and rate of phosphorus accumulation in the lake are calculated. The sum of those three is the net phosphorus release (if positive) or sedimentation (if negative). Rewriting equation (4-5):

$$\left(\sum_{i=t}^{t+\Delta t} Q_{in} P_{in} \right) \Delta t - \left(\sum_{i=t}^{t+\Delta t} Q_{out} P_{out} \right) \Delta t$$

$$- \left[P_L(t) V_L - P_L(t+\Delta t) V_L \right] = \text{net release or sedimentation}$$

where:

P_{in} = measured influent P concentration (M/L^3)

P_{out} = measured effluent P concentration (M/L^3)

Q_{in} = measured lake inflow (L^3/T)

Q_{out} = measured lake outflow (L^3/T)

P_L = measured lake water P concentration (M/L^3)

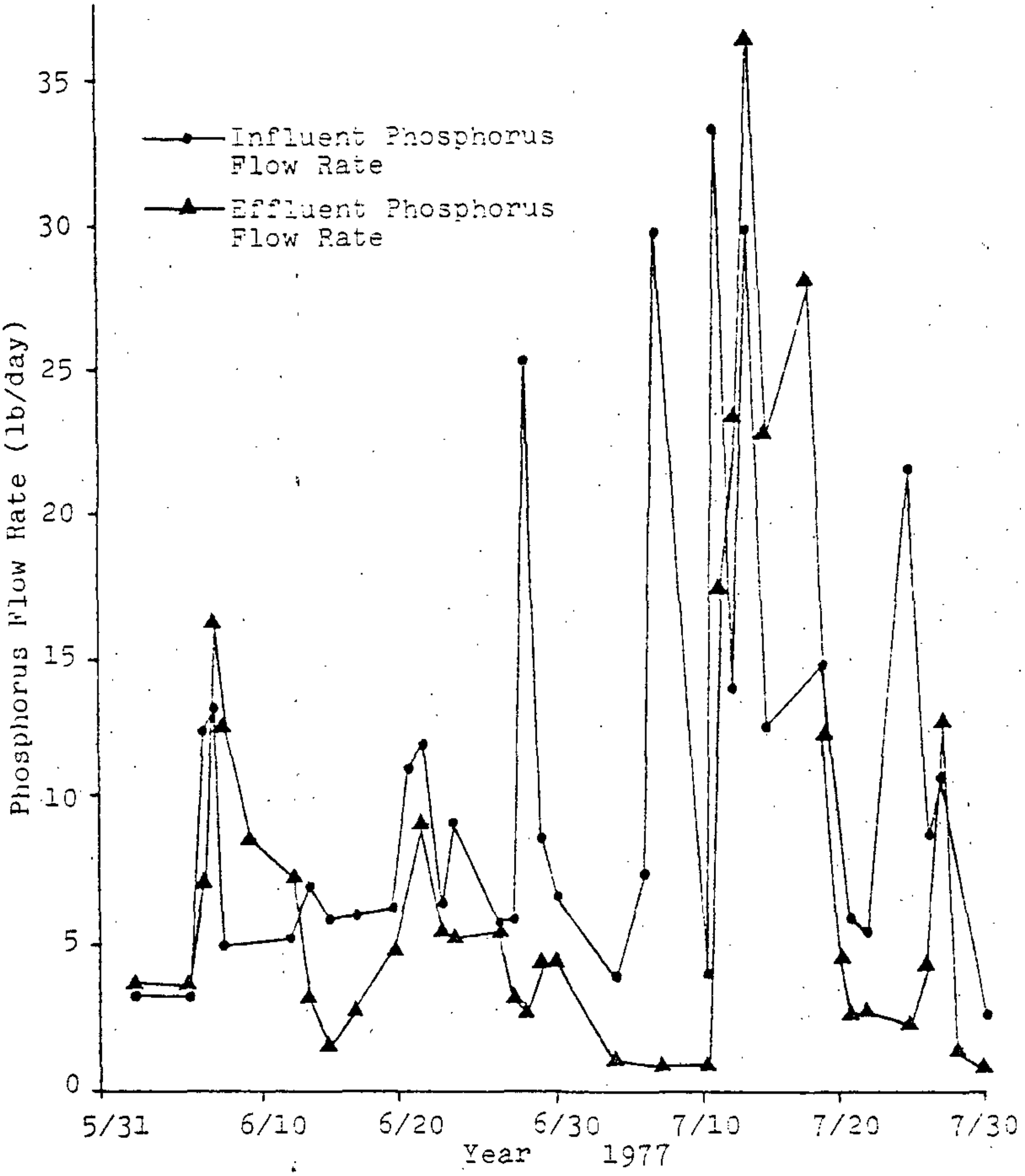

Figure 16. Variations of Influent and Effluent Phosphorus Flow Rate During June and July, 1977

i = number of measured data points during Δt

Figure 17 is a summary of the mass balance calculations. In the upper graph, the influent phosphorus load (positive) and the effluent load (negative) are both shown. In the middle graph, the accumulation of phosphorus in the lake water over the time step (either positive or negative) is depicted. The lower graph shows the balance of phosphorus and indicates the net reaction, either sedimentation or release, which has occurred.

The results of the mass balance support the concept that the sediments are now adsorbing phosphorus. Since very little release occurs during the time when it is most likely to occur, it must be concluded that the lake has recovered from the earlier, higher loading rates.

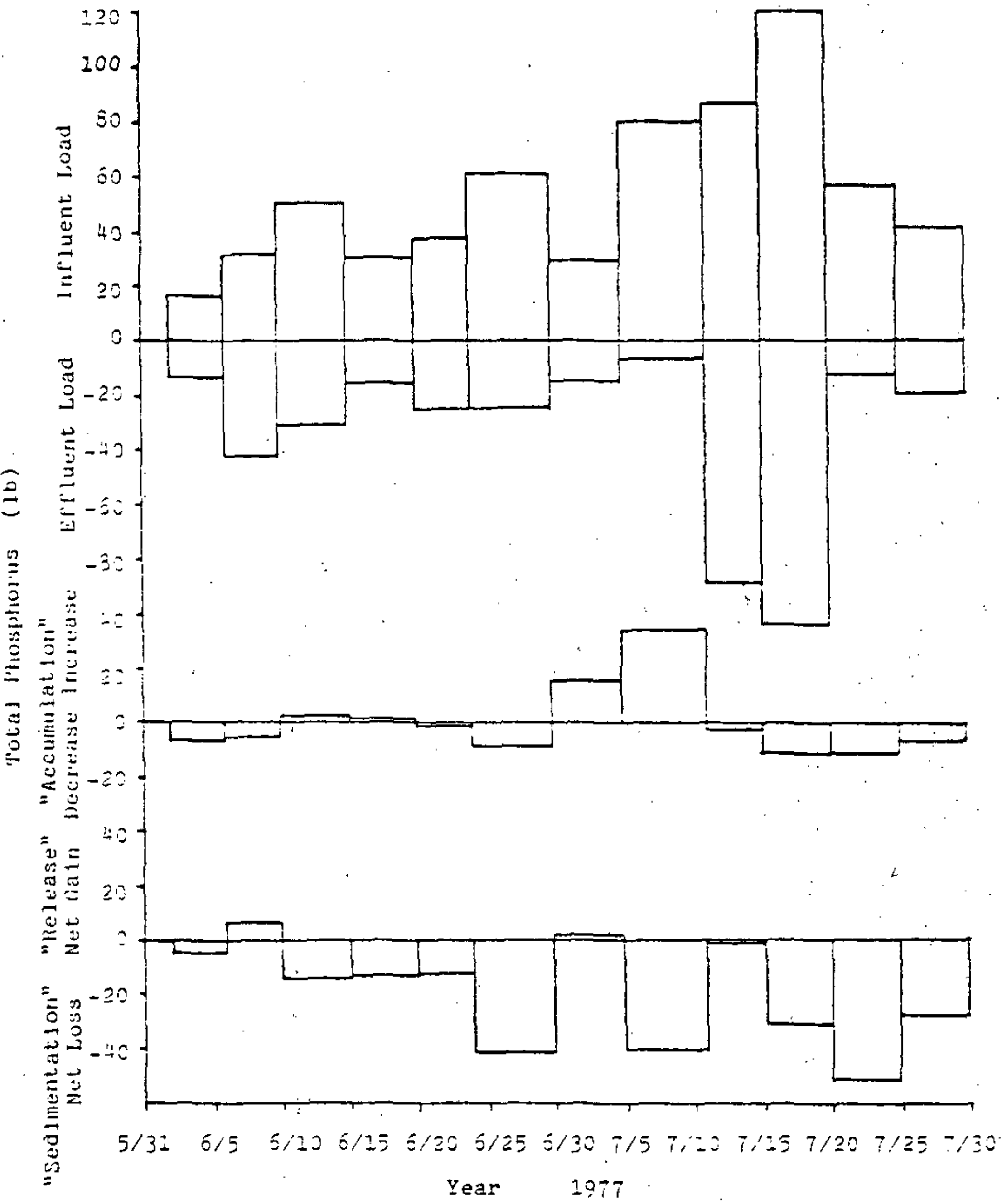

Figure 17. Phosphorus Mass Balance

CHAPTER V

MATHEMATICAL MODELING OF LAKE RECOVERY

A major purpose of this research is to test the model developed by Snow and DiGiano (1976) for Lake Warner, which has been presented in the literature survey, Chapter II.

The field measurements of this study have shown that the phosphorus concentration of Lake Warner dropped from an average of 90 μg/ℓ to 50 μg/ℓ in about one to two years after the discharge of wastewater from the Amherst Wastewater Treatment Plant was stopped in November 1975. In comparison with the prediction by Snow and DiGiano's model, as shown in Figure 18, it is evident that the recovery of the lake was more rapid than the predicted value of phosphorus concentration which should have been 73 μg/ℓ.

One major reason for the rapid recovery of the lake as compared to the predictions is the shift in phosphorus equilibrium between lake water and sediment which has since occurred. As shown in Figure 13, both the total and orthophosphate phosphorus of the Lake Warner sediment interstitial water decreased dramatically from 1974 to 1977, therefore the Snow and DiGiano findings must be modified. It is apparent that the shift of phosphorus equilibrium was the result of the reduction in discharge of organic matter to Lake Warner which, as a result, became

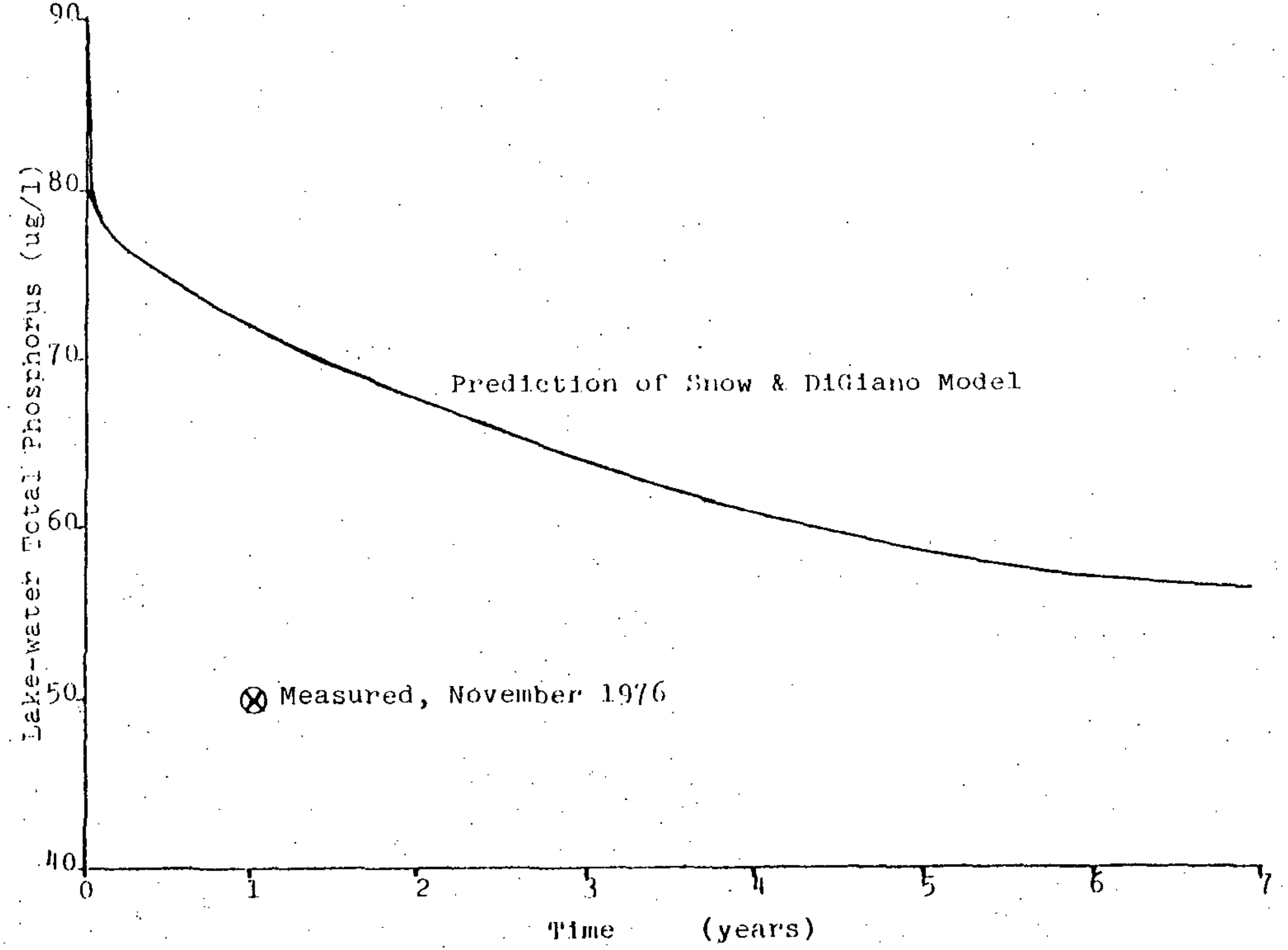

Figure 18. Comparison of Measured vs. Predicted Phosphorus Concentration in Lake Warner

well oxygenated. Under oxic conditions, the release of phosphorus from sediments was greatly reduced and net sedimentation thus resulted. A reliable forecast of these events would indeed be a difficult task, because they are dynamic phenomena, depending upon the pollutional discharges to a lake, and seasonal changes of oxidation-reduction potential, pH and temperature of lake water and sediments. The ideal approach would be to predict the dissolved oxygen in a lake and its relationship to the phosphorus equilibrium between lake water and sediments, as was found by Lung, Canale, and Freedman (1976). This, however would require the measurements of biochemical oxygen demand, algal content and its effect on dissolved oxygen, and other complex items which are beyond the scope of this study.

The following are two approaches to predict the rate of recovery from a eutrophic status of Lake Warner. The first is a simple approach based on a dilution model. The second is a rational engineering approach which is an extension of the Snow and DiGiano model.

A Simple Approach - Dilution Models

Dilution models were reviewed in Chapter II. Figure 19 shows a family of dilution curves for the Lake Warner recovery prediction, as well as the Snow and DiGiano prediction curve. The dilution curve in which $t_{rel} = 1$ is for the case that sedimentation and release of phosphorus

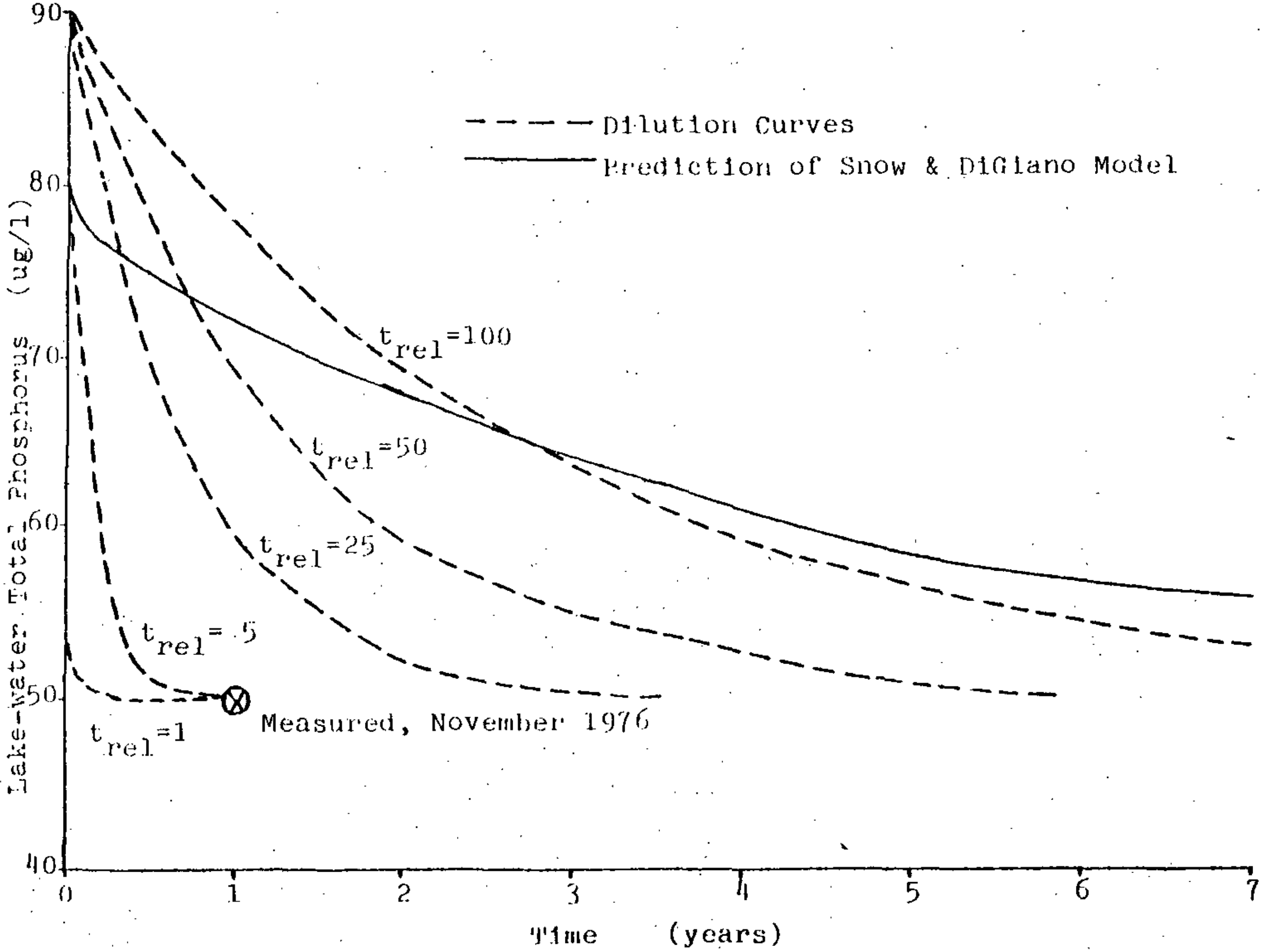

Figure 19. Comparison of Dilution Curves with Model Prediction

are exactly balanced. The dilution curve in which $t_{rel} = 5$ assumes sediment release is five times greater than sedimentation. The recovery prediction of Snow and DiGiano is roughly comparable to a dilution curve in which $t_{rel} = 100$, and therefore, their model predicts that phosphorus cycling processes cause phosphorus to remain in the lake 100 times longer than the hydraulic detention time. The reason for the long predicted recovery time by Snow and DiGiano is that they assume the linear non-variant empirical relationship between the solid-phase and interstitial-water phosphorus of the sediment as given in Equation (2-4) would remain unchanged. However the later field measurements discovered the fact that this relationship had been changing during the lake recovery.

A precise comparison of Lake Warner recovery time with the dilution curves as given in Figure 19 cannot be made, since the recovery was so rapid that it had already occurred prior to the measurements made in this study. However, roughly as shown in Figure 19, t_{rel} could be on the order of 1 to 5.

Ahlgren (1977) found the recovery of Lake Novikken approximated to the dilution curve of t_{rel} equal to 1.3. It was pointed out that the dilution curves only predicted the annual mean values, but not the seasonal variations. Concentrations were higher than the mean in summer, during conditions favorable to phosphorus release from sediment, and lower in spring due to rapid sedimentation of phosphorus after the spring diatom bloom.

Lake phosphorus mass balance calculations coupled with dilution models are useful as a first estimate of lake recovery time. In particular, lakes with small hydraulic detention times or large flushing capacity, such as Lake Warner, would be most likely to show rapid responses in recovery to changes in influent loading, as shown in the analysis of the data collected in this study by means of mass balance (see Chapter IV). It was indicated that net seidmentation occurred through June and July 1977. This corresponds to a t_{rel} less than 1. However, the dilution technique does not account specifically for actual lake processes which have occurred, but simply gives an estimate of the average lake response due to a change of pollutional loading. It is desirable to develop a model with a rational engineering approach to reflect not only a gross response, but also the phosphorus-cycling processes involved.

A Rational Engineering Approach

The development of a rational engineering model for lake recovery will be based upon the work of Snow and DiGiano. First, a sensitivity analysis will be performed to observe the effect on model results of varying certain input parameters. Subsequently, a modification of the Snow and DiGiano model will be proposed which will improve its flexibility without basically altering its rationale.

The sensitivity of the model to some selected input parameters is presented in Figures 20,21,22,23, and 24. The results are summarized in Table 3 as percent change in lake phosphorus concentration at selected times resulting from a 10 percent change in input parameters. The parameters chosen for this analysis are those which cannot be easily measured in the field and are therefore most subject to error. These are the phosphorus release rate from sediment, phosphorus sedimentation coefficient, sediment reactive depth, percent exchangeable phosphorus, and the empirical relationship between interstitial and solid-phase phosphorus. Parameters such as detention time, lake surface area and volume can be measured fairly accurately and therefore will not be included.

In general, the phosphorus concentration profiles in Figures 20 through 24 resulting from varying the input parameters look similar except for Figure 22, the sediment reactive depth. In Table 3, the largest percent differences occur in the early part of the simulation, and then decrease with time, in all cases except the sediment reactive depth. The reactive depth percent difference increases slowly with time, instead. Adjustment of the exchangeable fraction of sediment phosphorus causes the greatest deviation. All of the percent differences in the lake water phosphorus concentration computed are less than the 10 percent variation of the input parameters.

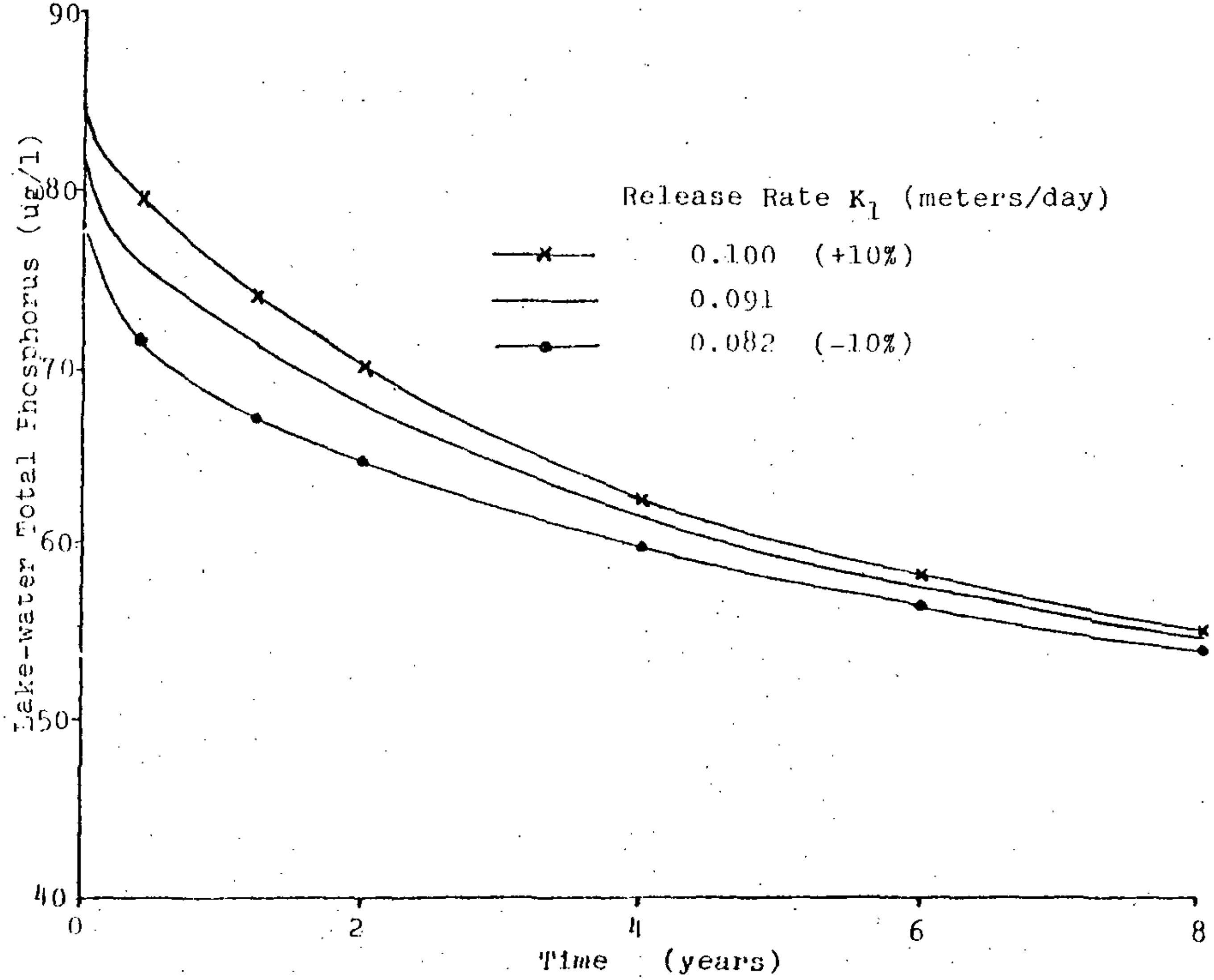

Figure 20. Sensitivity of Release Rate K_1

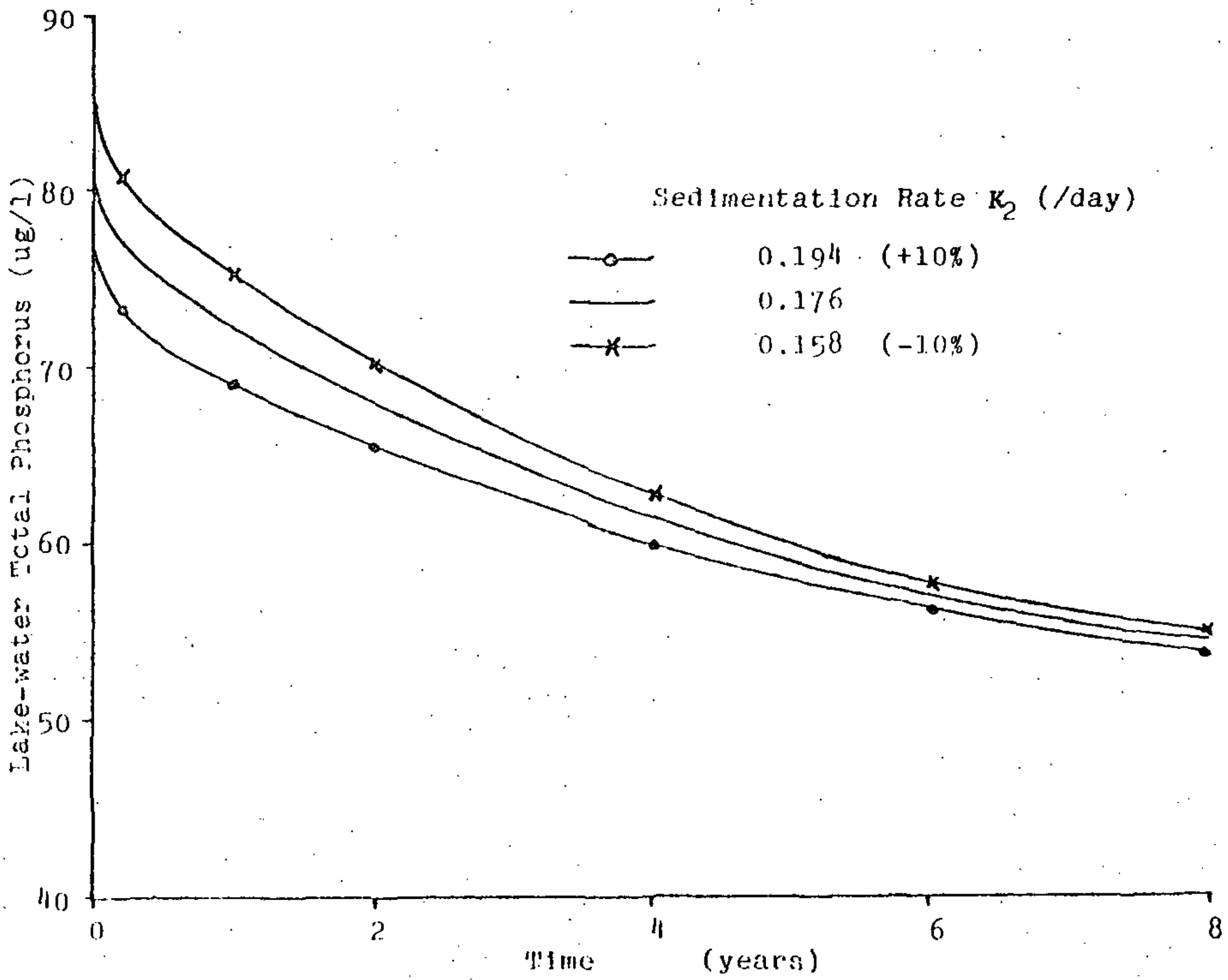

Figure 21. Sensitivity of Sedimentation Coefficient, K_2

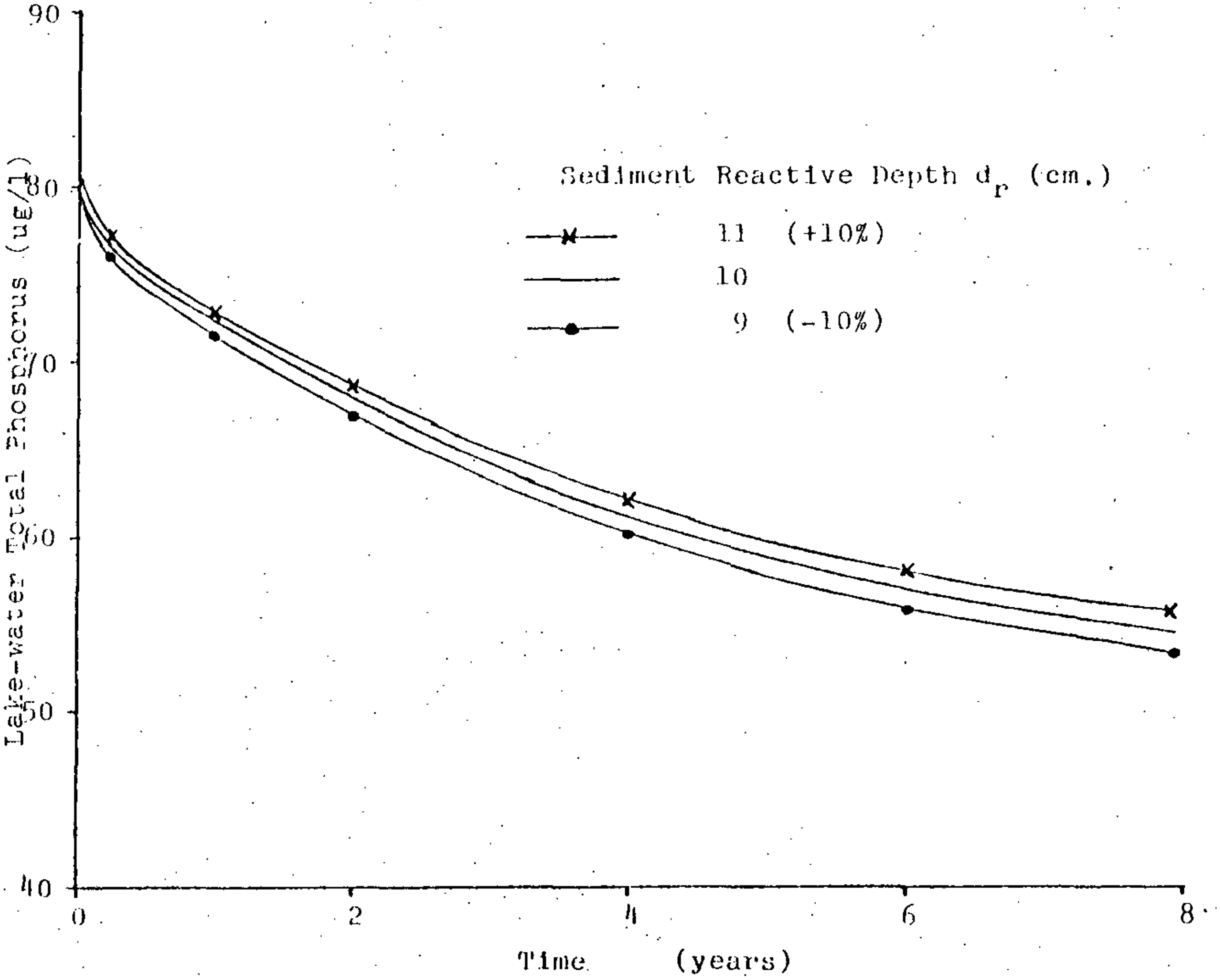

Figure 22. Sensitivity of Sediment Reactive Depth d_r

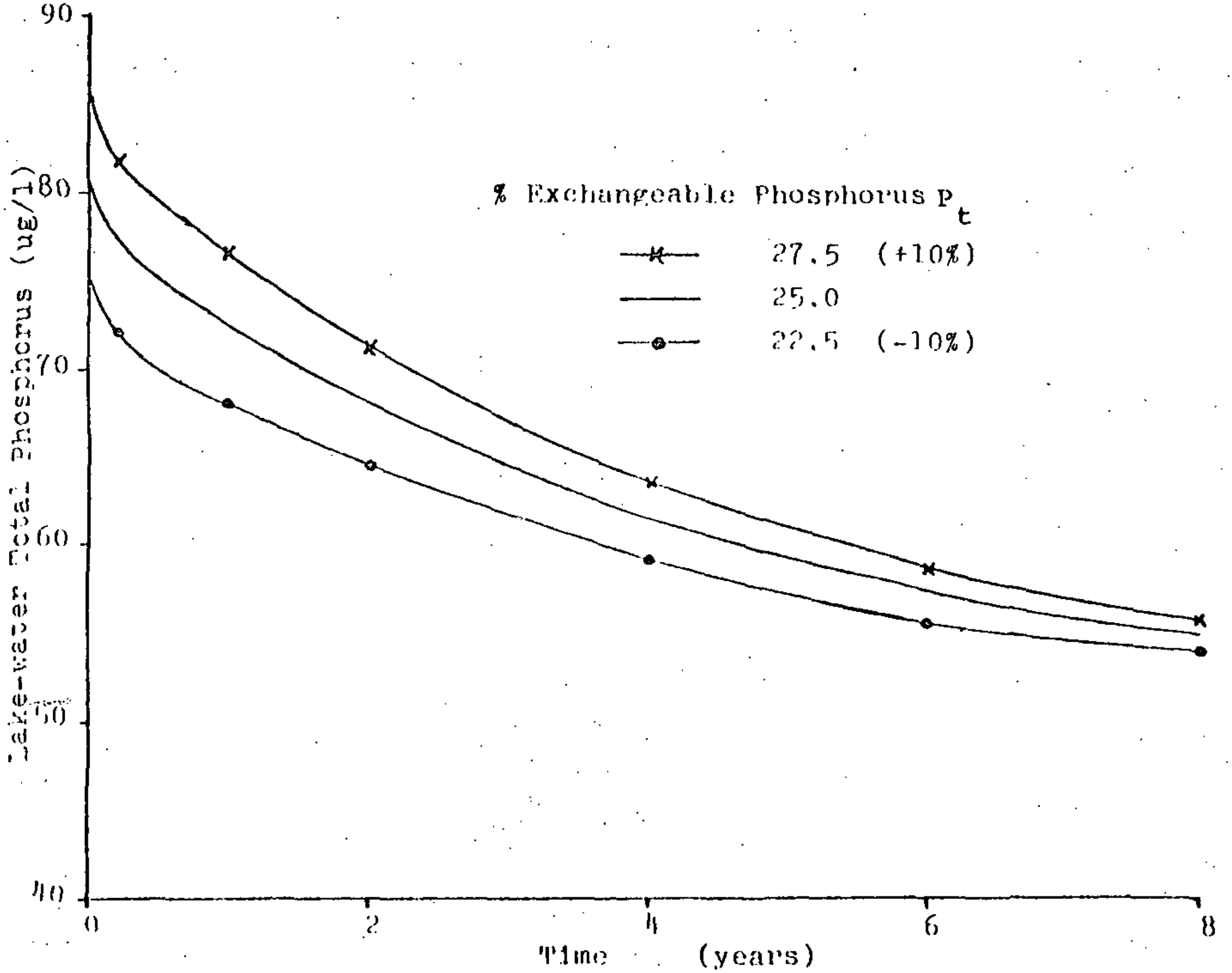

Figure 23. Sensitivity of Percent Exchangeable Phosphorus P_t

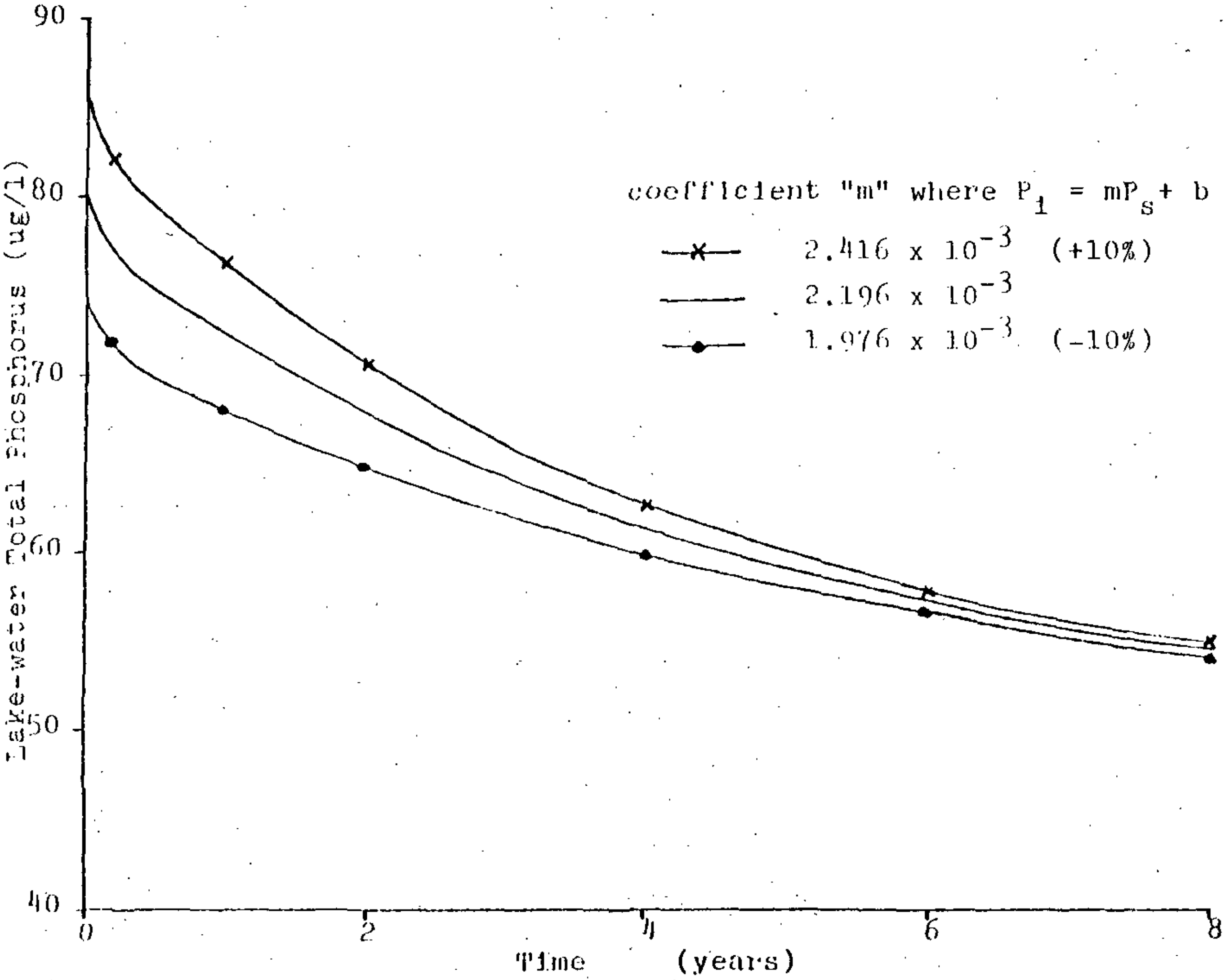

Figure 24. Sensitivity of Interstitial vs. Solid-phase Sediment Phosphorus Relationship $P_i = mP_s + b$

Table 3

Sensitivity Analysis

Time (days)	Time (yrs)	% Change in Lake Phosphorus Concentration Resulting from 10 Percent Change in Stated Parameter K_1	K_2	d_r	P_t	m
1		1.56	1.52	0.00	1.74	1.74
146	2/5	5.76	4.85	0.30	6.46	6.19
365	1	4.93	4.21	0.70	5.90	5.26
730	2	3.72	3.28	1.18	5.02	3.93
1825	5	1.32	1.35	1.71	2.95	1.29

where:

K_1 = release rate (meters/day)

K_2 = sedimentation rate (1/day)

d_r = sediment reactive depth (centimeters)

P_t = percent exchangeable phosphorus

m = interstitial vs. solid-phase sediment coefficient where $P_i = mP_s + b$

The sedimentation and release coefficients were determined in situ by Snow and DiGiano by a procedure outlined in Chapter II of this report. Based on a release rate of 0.091 m/day and a sedimentation rate of 0.176 ℓ/day the steady-state value of interstitial phosphorus is 245 μg/ℓ under the assumption that the lake water phosphorus is reduced from 90 to 50 μg/ℓ. The steady-state value of 245 μg/ℓ compares extremely well with the measurements in this study (see Figure 13).

Adjustment of the sediment reactive depth causes a difference which grows with time. This parameter defines the total mass of phosphorus which is available for interaction with the lake water. Since it is difficult to determine accurately, a range of values should be investigated. Long-term simulations are most likely subject to error.

The exchangeable fraction limits the amount of sediment solid-phase phosphorus available for conversion to interstitial phosphorus. After the conversion, diffusion to the lake water controls the mass transfer. Thus, the lake water phosphorus concentration profiles in Figure 23 become more gentle in gradient as the time increases. Error in specification of this parameter would be greatest in the early years of recovery, and therefore, particular caution should be exercised for short-term simulations. Since the exchangeable fraction is difficult to estimate and is dependent on environmental factors such as redox potential (Ku, 1975), in order to make the best of the calculations, the values should be expressed in a range.

The empirical relationship between interstitial and solid-phase phosphorus has been shown to be highly variable in Lake Warner (see Figure 14). The assumption made by Snow and DiGiano that this relationship is linear and remains constant is the main reason that the Snow and DiGiano model predictions are not applicable. According to Snow and DiGiano, a linear equilibrium relationship exists between the solid and liquid phase phosphorus of sediments, because the equilibrium is assumed to be controlled by adsorption, a physical-chemical process. However, Stumm and Leckie (1971) stated that bacteria convert as much organic solid-phase phosphorus into soluble interstitial phosphorus as is released by physical-chemical processes. Lung, Canale, and Freedman (1976) outlined several processes which affect the phosphorus transformations in lake sediments, such as oxidation and breakdown of organic sediments, physicochemical adsorption, precipitation and dissolution to and from mineral forms, and both chemical and biological oxidation and reduction of inorganic species. Since all these processes are involved, and difficult to be evaluated individually, the following approach is proposed whereby these processes are lumped together and described by one reaction coefficient.

In the proposed approach, the Snow and DiGiano model is extended to include a mass balance on the sediment interstitial phosphorus. This would avoid the use of the linear and constant relationship between the sediment interstitial and solid-phase phosphorus concentrations. The following mass balances are suggested for the lake water, sediment solid-phase phosphorus, and sediment interstitial phosphorus:

Sediment Solid-Phase:

$$\underset{\text{mass accumulation}}{V_s \frac{dP_s}{dt}} = \underset{\text{sedimentation from lake water}}{K_2 P_L V_L} - \underset{\text{conversion to interstitial phosphorus}}{K_3 P_S V_S} \qquad (5\text{-}1)$$

Sediment Interstitial Water:

$$\underset{\text{mass accumulation}}{\varepsilon V_s \frac{dP_i}{dt}} = \underset{\text{conversion from sediment solid-phase to liquid-phase}}{K_3 P_S V_S} - \underset{\text{diffusion to lake water}}{\varepsilon A K_1 (P_i - P_L)} \qquad (5\text{-}2)$$

Lake Water:

$$\underset{\text{mass accumulation}}{V_L \frac{dP_L}{dt}} = \underset{\text{mass in}}{QP_o} - \underset{\text{mass out}}{QP_L} + \underset{\text{release}}{\varepsilon K_1 A (P_i - P_L)} - \underset{\text{sedimentation}}{K_2 P_L V_L} \qquad (5\text{-}3)$$

where

$V_s = D_r A$ = reactive volume of sediment (L^3)

K_3 is the reaction coefficient describing conversion rate between particulate and dissolved sediment phosphorus (1/T) The lake water phosphorus mass balance is the same as that proposed by Snow and DiGiano which was presented as Equation (2-1). The sediment solid-phase and interstitial-water phase mass balances include a new coefficient K_3. This coefficient describes the rate of conversion of solid-phase to dissolved phosphorus in the sediment. A negative value of K_3 would indicate a net conversion of dissolved to particulate phosphorus in the sediment.

No experimental evidence exists to support the use of a particular value for K_3. Lung, Canale, and Freedman (1976) calibrated a mathematical model for White Lake, however, which included a rate coefficient for the conversion of solid-phase to dissolved phosphorus in sediments. In Table 4, various values of the coefficient K_3 which were used by Lung and Canale (1977) are presented. Obviously, the values of K_3 would vary seasonally and depend upon environmental factors, such as dissolved oxygen, redox potential, pH and temperature.

Equations (5-1), (5-2), and (5-3) can be written in finite difference form as follows:

$$P_s(t+\Delta t) = P_s(t) + \left[\frac{K_2 P_L(t) V_L}{D_r A} - K_3 P_s(t) \right] \Delta t \qquad (5\text{-}4)$$

$$P_i(t+\Delta t) = P_i(t) + \left\{ \frac{K_3}{\varepsilon} P_s(t+\Delta t) - \frac{K_1}{D_r} [P_i(t) - P_L(t)] \right\} \Delta t \qquad (5\text{-}5)$$

$$P_L(t+\Delta t)$$

$$= \frac{P_o + (\frac{\varepsilon A}{V_L})\ K_1 P_i(t+\Delta t)\ \bar{T}}{1 + (\frac{\varepsilon A}{V_L})\ K_1\bar{T} + K_2\ \bar{T}}\ \{1\text{-}\exp\ [-(\frac{1}{\bar{T}} + \frac{\varepsilon A}{V_L}\ K_1 + K_2\ t]\} \tag{5-6}$$

$$+\ P_{L_o}\ \exp\ [-\ (\frac{1}{\bar{T}} + \frac{\varepsilon A}{V}\ K_1\ +\ K_2)\ t]$$

Note that equation (5-6) is identical to equation (2-5).

In Figure 25, the predictions of the Snow and DiGiano model and the new model are compared. The K_3 value in the new model was adjusted until the simulation matched closely with that of Snow and DiGiano's model. The resulting value of K_3 was 1.86×10^{-3} per day, which is over twice as high as the values presented in Table 4 for anaerobic conditions in White Lake. The new model simulations are also shown using a K_3 value of 1×10^{-3}/day. This value, selected through model calibration, seems reasonable for the data collected in Lake Warner. Even using the K_3 value which applies to anaerobic conditions in White Lake, the Lake Warner recovery predicted by the new model is shown to be much more rapid than the recovery simulated by Snow and DiGiano. Since K_3 is assumed to be a first order reaction, use of a single K_3 value is comparable to use of the linear relationship developed by Snow and DiGiano (1976). However, programming advantages are gained by allowing a variable K_3 rather than a family of linear equilibrium relationships.

Table 4

Values of K_3

Reference	Location	K_3 (/day)	Comment
Lung, Canale, and Freedman (1976)	White Lake	0×10^{-4}	Aerobic
Lung, Canale, and Freedman (1976)	White Lake	8×10^{-4}	Anaerobic
Lung and Canale (1977)	White Lake	1.4×10^{-4}	Annual Average
Lung and Canale (1977)	White Lake	2.0×10^{-5}	Aerobic
Lung and Canale (1977)	White Lake	7.9×10^{-4}	Anaerobic

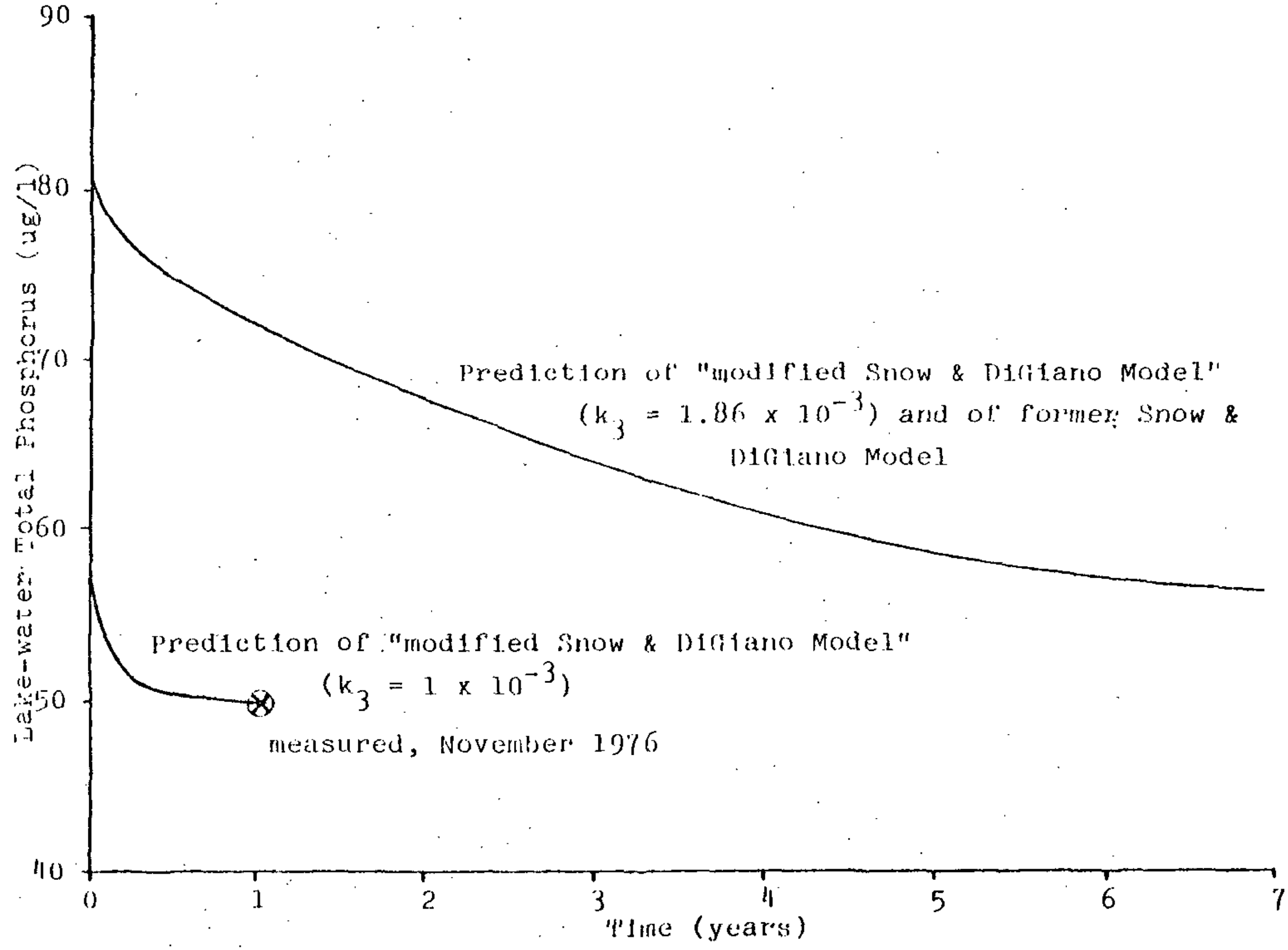

Figure 25. Comparison of "Modified Snow & DiGiano Model" and former Snow & DiGiano Model

Summary

The model developed by Snow and DiGiano (1976) has been extended to include a mass balance on phosphorus in the sediment interstitial water. This modification eliminates the need for specifying a constant linear equilibrium relationship between sediment interstitial and solid-phase phosphorus which is based on adsorption-desorption. Instead, a first order reaction is assumed between sediment solid-phase and interstitial phosphorus. This assumption lumps all chemical, physical, and biological factors together into one reaction rate coefficient. Although no experimental evidence and limited comparison between prediction and field data exist to support the use of this technique, it is thought that this method is more flexible and practical for the following reasons:

1. Inclusion of a mass balance for the sediment interstitial phosphorus is a forward step towards a "rational" engineering model.
2. The relationship between sediment interstitial and solid-phase phosphorus includes all biological, chemical and physical transformation processes, and probably is not linear and constant for most cases.
3. Existing field data may be utilized to arrive at an estimate of K_3 through model calibrations.

4. Field data collection programs may be readily modified for the purpose of estimating K_3 through model calibration.

Several assumptions made by Snow and DiGiano have been followed in this work. One is that the lake is shallow, or has one layer with no vertical concentration gradients. Another assumption is that total phosphorus in the lake water includes both dissolved and particulate forms. Knowledge of the reaction coefficients, such as algal uptake of dissolved phosphorus and settling of particulate phosphorus and a data collection program which includes sampling of both forms, would allow expansion of the model. A third assumption is that the lake acts as a completely-mixed reactor. This is in line with all research reviewed in this report, and should be adequate for most lakes except in special cases where longitudinal or lateral gradients are judged to be important. Mathematically, the proposed model could easily be extended to cover most of these situations. The real difficulty involved is the determination of any additional reaction coefficients which would be included.

A simple dilution model approach and a more complex rational engineering approach to help address questions of lake recovery time after reduced phosphorus loadings have been outlined. The key to the utilization of either or both approaches is a well-designed field data collection program, particularly the mass balances of influent, effluent, and lake water and sediment phosphorus, which are needed to define the net rate of sedimentation or release. These measurements should be taken at daily or shorter intervals, and should be more intensive during rainstorms. Because of the cost of such a sampling program, it may be sufficient to collect data in a minimum of two selected months during the year (a summer month and a winter month). Secondly, the long-term monitoring of phosphorus in the lake water, influent and effluent, and sediment solids and interstitial water should be conducted at less frequent intervals, weekly or even monthly. Such data of long records are needed for the final verification or formulation of mathematical models for developing lake management plans.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

A sampling program was carried out to monitor the effect of a reduction in phosphorus loading on Lake Warner in Hadley, Massachusetts. It was found that lake water was reduced from the average phosphorus concentration of the 90 μg/ℓ in 1974 to 50 μg/ℓ in 1977. This occurrence was probably due to the elimination of the wastewater discharge from Amherst to the lake in November 1975. Further reduction in phosphorus concentration (below 50 μg/ℓ) is not likely because the agricultural runoff would maintain a background concentration close to this value.

An algal bloom of one week duration was observed in 1977 which occurred after a period of low flow and high temperature. Apparently, a phosphorus concentration of 50 μg/ℓ was high enough to sustain a bloom under the proper environmental conditions. However, a significant improvement has been visible in comparison with the lake status in 1973 and 1974 when Snow and DiGiano (1976) described the lake as "highly eutrophic with extreme bloom proportions of blue-green algae". This visible improvement was confirmed in converations with local residents.

Mass balance calculations based on the June and July 1977 data, have shown that net phosphorus sedimentation occurred during this period. That was a summer period with low flows and high temperatures which created conditions most favorable to phosphorus release from the sediments. But there was no evidence that phosphorus was released from sediments under such favorable conditions. It may be surmised that Lake Warner was recovering from its previous pollutional loads and that the sediments were no longer a significant source of phosphorus. These mass balance calculations made from intensive (daily) sampling over a short period (one month, for example) have proved extremely effective in determining the role of the sediments in lake phosphorus cycling.

Using data collected during this study, two methods of predicting lake recovery were tested. The first was a simple dilution approach, in which the ratio of phosphorus sedimentation to sediment phosphorus release determines the length of time necessary for a given change in phosphorus concentration. Lake Warner had returned to its natural conditions prior to the 1977 sampling program so that this ratio could not be determined by field measurements, but it was calculated that phospohrus release was less than five times sedimentation. The findings do not agree with model claculations made by Snow and DiGiano (1976) which indicated that release would be about one hundred times greater than sedimentation.

One of the reasons for the difference between the Snow and DiGiano predictions and measured data could be the inclusion of a linear and non-variant relationship between interstitial and solid-phase sediment phosphorus in their model. Data in this study indicated that the equilibrium relationship between these two parameters had shifted, probably due to the elimination of the wastewater discharge to the lake. As a result dissolved oxygen concentrations at the lake bottom were increased and therefore oxidation-reduction potentials were raised. This shift was not accounted for in the Snow and DiGiano model.

A modification was proposed to moidfy the model by including a mass balance equation on the interstitial phosphorus and a new reaction rate describing the net rate of conversion between interstitial and solid-phase phosphorus. Although this reaction rate has not been determined through laboratory experiments, model calibration results are comparable with similar calibrations by Lung, Canale, and Freedman (1976), and Lung and Canale (1977).

Several recommendations which have arisen out of this work are presented as follows: first, in designing of field sampling program for monitoring lake recovery, short-term intensive programs can be as effective as the usual long-term programs. As previously illustrated, the role of the sediments can be delineated by a short-term program to have a direct and significant impact on the length of time a lake will take to recover. The simple dilution model may be used as a follow-up to the suggested field program for an idea of the length of recovery time. This is, of course, assuming that the ratio of release to sedimentation remains constnat.

The more complex "modified Snow and DiGiano model" must be tested on other lakes. In particular, in situ determinations of reaction rates such as those used by Snow and DiGiano (1976) in Lake Warner are recommended. The validity of the sedimentation and release rates measure by their field work was suggested in this study by comparing predicted steady-state concentrations of interstitial sediment phosphorus with the measured values. It is desirable to include in their field test the measurements of sediment phosphorus. Such additional measurements would enable the estimation of the net conversion rate between interstitial and solid-phase phosphorus proposed as a result of this study. Increased knowledge of the reaction rates would eventually lead to the ability to use time varying rates to describe lake recovery under both anoxic and oxic conditions.

It is strongly urged that these predictive tools be studied further and applied wherever possible. These tools are invaluable to engineers and planners who must assess alternative control strategies of wasteload reduction in controlling lake eutrophication.

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APPENDIX A

MILL RIVER AND LAKE WARNER WATER QUALITY

Table 5

Mill River Water Quality

Date	Location	Total P (µg/ℓ)	Ortho-P (µg/ℓ)	Flow (cfs)
10/13/76	I[1]	57		
	O	64		
10/20/76	I	39		
	O	40		
10/21/76	I	145		
	O	86		
11/3/76	I	36	23	
	O	44	18	
11/10/76	I	60	12	
	O	35	9	
11/16/76	I	35		
	O	35	5	
11/24/76	I	25	1	
	O	30	3	
12/1/76	I	30	3	
	O	37	3	
12/7/76	I	210	55	
	O	90	30	
12/16/76	I	31	9	
	O	50	16	
12/22/76	I	40	12	24
	O	37	11	
12/30/76	I	43	16	14
	O	45	20	
1/4/77	I	60	15	10
	O	45	11	
1/12/77	I	107	47	21
	O	68	16	
2/2/77	I	64	47	12
	O	36	30	8

Table 5. Mill River Water Quality (Continued)

Date	Location	Total P (μg/ℓ)	Ortho-P (μg/ℓ)	Flow (cfs)
2/7/77	I	50	13	
	O	48	11	
2/14/77	I	112	30	23
2/16/77	I	69	42	
	O	46	15	
2/24/77	I	31	23	11
	O	39	27	15
2/25/77	I	143	30	61
2/28/77	I	227	76	141
3/2/77	I	93	35	57
	O	163	104	69
3/10/77	I	40	15	
	O	42	21	
3/14/77	I	65	29	227
	O	82	49	
3/23/77	I	63	34	230
	O	57	30	
4/5/77	I	45	20	170
	O	40	8	
4/15/77	I	31	5	41
	O	100	5	
4/25/77	I	34	6	276
	O	68	25	
5/5/77	I	30	11	60
	O	53	14	
5/9/77	I	63	8	210
	O	60	30	
5/25/77	I	33	18	25
	O	36	14	32

Table 5. Mill River Water Quality (Continued)

Date	Location	Total P (µg/ℓ)	Ortho-P (µg/ℓ)	Flow (cfs)
5/31/77	I	18	16	
6/6/77	I	41	21	15
	O	43	18	15
6/7/77	I	69	52	33
	O	61	33	23
6/7/77	I	49	25	51
	O	49	27	63
6/8/77	I	20	18	46
	O	51	27	45
6/10/77	I		26	48
	O	53	29	31
6/13/77	I	31	18	[illegible]
	O	59	14	25
6/14/77	I	47	15	29
	O	52	20	11
6/15/77	I	41	25	27
	O	37	14	7
6/17/77	I	46	44	25
	O	58	16	9
6/20/77	I	40	20	29
	O	40	14	23
6/21/77	I	50	20	42
6/22/77	I	59	38	38
	O	59	20	29
6/23/77	I	41	25	27
	O	46	12	19
6/24/77	I	69	58	25
	O	58	21	13
6/27/77	I	44	20	25
	O	52	16	21

Table 5. Mill River Water Quality (Continued)

Date	Location	Total P (μg/ℓ)	Ortho-P (μg/ℓ)	Flow (cfs)
6/28/77	I	52	33	21
	O	44	18	13
6/29/77	I	225	192	21
	O	46	20	11
6/30/77	I	65	33	25
	O	54	12	15
7/1/77	I	60	40	21
	O	48	20	17
7/5/77	I	46	24	15
	O	57	13	1
7/7/77	I	91	31	15
	O	57	12	3
7/8/77	I	191	115	29
	O	62	24	1
7/11/77	I	49	26	17
	O	47	16	1
7/12/77	I	93	55	66
	O	58	27	55
7/13/77	I	66	25	39
	O	76	24	57
7/14/77	I	77	25	72
	O	74	24	91
7/15/77	I	61	60	38
	O	93	25	45
7/18/77	I	88	50	96
	O	56	32	93
7/19/77	I	38	37	72
	O	50	26	47
7/20/77	I	49	41	28
	O	54	42	15
7/21/77	I	47	39	23
	O	51	20	9

Table 5. Mill River Water Quality (Continued)

Date	Location	Total P (µg/ℓ)	Ortho-P (µg/ℓ)	Flow (cfs)
7/22/77	I	43	38	23
	O	59	20	9
7/25/77	I	114	64	35
	O	80	29	5
7/26/77	I	54	28	29
	O	58	25	13
7/27/77	I	100	45	20
	O	52	31	45
7/28/77	I	82	28	1
7/29/77	I	47	45	16
	O	67	36	1
8/1/77	I	34	24	15
	O	49	15	1
8/2/77	I	54	38	117
	O	95	51	217
8/3/77	I	41	35	45
	O	98	51	69
8/17/77	I	29	21	
	O	85	76	
8/29/77	I	71	41	
	O	65	21	
9/9/77	I	71	44	
	O	22	11	
9/14/77	I	67	59	
	O	21	20	
9/16/77	I	54	31	
	O	64	28	
9/27/77	I	71	31	
	O	200	125	

TABLE 6. LAKE WARNER WATER QUALITY

Date	Sampling location	Total P (ug/l)	Ortho-P (ug/l)	Inter Total P$^{(1)}$ (ug/l)	Inter-Ortho-P$^{(2)}$ (ug/l)	Dissolved oxygen		Temperature		Depth (ft)	Secchi-depth (ft)	pH	Alkalinity (mg/l CaCO$_3$)	Turbidity (JTU)	NH$_3$-N (mg/l)	NO$_3$-N (mg/l)
						Surface (mg/l)	Bottom (mg/l)	Surface (deg C)	Bottom (deg C)							
10/13/76	1	40				7.7	7.5	14	12			7.0	24	1.4	0.30	
	2	47				7.3	7.3	10	9			7.2	23	1.5	0.55	
	3	45				7.1	7.2	11	10			6.7	24	2.5	0.37	
	4	60				6.5	6.3	13	12.5			6.8	25	4.4	0.41	
11/16/76	2	38	7			12.8		4				6.3	24	1.5	0.16	
	3	36	6			13.2		4				6.6	22	1.5	0.17	
	4	33	6			12.7		4				6.4	21	1.5	0.18	
1/4/77	3	51		250	150	13.5		1				6.5	16	1.5	0.23	
	4	39	9	260	150	13.4		1				6.5	15	1.5	0.23	
2/16/77	1	42	13					0				6.5	28	2.7	0.90	5.9
	2	72	20					0				6.4	20	3.6	1.06	2.7
	3	56	15					0				6.5	19	2.9	0.60	1.9
	4	46	15					0				6.3	17	2.0	0.42	1.8
4/15/77	1	78	11			10.8	10.8	10		6	6	6.5	14		0.13	1.8
	2	43	13			9.6	9.6	11.5		5	5	6.4	14	1.5	0.14	1.6
	3	33	12			10.4	9.6	13.5		4	4	6.4	14	3.5	0.11	1.4
	4	28	9			10.8	10.8	13		4	4	6.4	15	2.5	0.09	1.6
5/31/77	1	55	23			9.5	8.7	21	17	4.75	4.75	6.6	19	2.7		
	2	89	39			8.9	9.2	22	19	3	2.5			5.4		
	3	50	29			10.3	8.6	24	15	6.5	3			3.6		
	4	45	25			10.0	8.5	24	15	9.5	2.5			3.3		
6/10/77	1	48	31	280	116	12.0		12		4						
	2	52	41	218	129	9.0		13.5		3	2.5					
	3	67	36	214	72	8.6		16		6						
	4	49	25	98	40	8.0		16		8.75	3					
6/15/77	1	52	36	234	95	7.1	7.5	19		5	3					
	2	54	38	83	45	8.9	8.2	21		4.5	3.5					
	3	51	16	151	71	9.6	8.6	21.5		6.75	4					
	4	40	11	125	42	10.0	9.5	23		8.75	4.5					
6/23/77	1	54	31	136	72	6.7	7.0	18	17	3.5	3	6.7	24		0.29	1.4
	2	56	22	152	84	7.0	7.0	21	19	6	2	6.7	24		0.29	1.0
	3	52	18	152	54	7.6	6.8	22	21	6	3	7.1	22		0.32	1.0
	4	70	10			8.5	5.2	23	20	8.75	5	7.1	20		0.16	1.2

TABLE 6 (Continued). LAKE WARNER WATER QUALITY

Date	Sampling location	Total P (ug/l)	Ortho-P (ug/l)	Inter Total P(1) (ug/l)	Inter-Ortho-P(2) (ug/l)	Dissolved oxygen Surface (mg/l)	Dissolved oxygen Bottom (mg/l)	Temperature Surface (deg C)	Temperature Bottom (deg C)	Depth (ft)	Secchi-depth (ft)	pH	Alkalinity (mg/l CaCO$_3$)	Turbidity (JTU)	NH$_3$-N (mg/l)	NO$_3$-N (mg/l)
6/30/77	1	46	26	153	48	6.5	6.5	21.5	22	6.5	4					
	2	45	14	193	79	9.3	9.0	25.5	26	3	2.5					
	3	36	10			9.5	9.5	25	25	6.5	4					
	4	40	9	167	211	9.2	9.0	25	24	9	4.5					
7/7/77	2	68	17	580	119					5.75	1.5					
	3	76	13	267	150			21		6.5	2					
	4	51	10	157	85			21		9	3					
7/13/77	1	80	40	325	85	9.0	8.7	23	22	4.5	2.5					
	2	109	4	277	76	13.8	7.0	25	22	4.5	1.5					
	3	107	7	315	53	18.0	18.0	26.5	25	5	1.5					
	4	88	16	170	75	23.0	15.3	25	16	8.5	2					
7/28/77	1	35	30		33											
	2	69	26		190											
	3	69	24		75											
	4	64	25		280											
8/29/77	1	55	29	160	38	8.5	7.5	24	22	5	3.5	6.9	25	2.0	0.10	2.0
	2	66	11	206	69	12.0	10.5	26	24.5	4	3	9.2	26	2.8	0.16	0.5
	3	49	9	258	33	14.0	14.0	26	26	4.5	4	9.5	25	2.2	0.10	0.4
	4	44	10	320	59	15.0	15.8	26	23.5	9	3	9.5	24	2.3	0.10	0.4
9/27/77	1	125	50													
	2	110	54													
	3	140	68													
	4	130	98													

1. Total phosphorus in interstitial sediment water.
2. Ortho-phosphorus in interstitial sediment water.

APPENDIX B

STREAM GAGING DATA

Table 7. Stream Gaging Results at Lake Warner Inlet

Date (1977)	Stage (ft)	X-Section Area (sq ft)	Average Velocity (fps)	Flow (cfs)
2/2	0.67	19.7	0.62	12.2
2/11	0.67	18.8	0.68	12.7
2/14	0.90	22.1	1.06	23.4
2/24	0.71	19.5	0.54	10.6
2/28	2.6	68.3	2.06	141
3/2	1.5	40.7	1.40	57.1
3/11	2.4	68.6	1.68	115
3/14	4.5	210	.92	194
4/1	2.1	55.9	1.68	94.2
4/6	2.8	75.8	1.83	139
4/15	1.2	35.5	1.14	40.6
4/29	1.03	29.6	0.91	27.0
5/25	1.05	29.2	0.85	24.8
6/6	0.80	21.7	0.71	15.4
6/20	1.00	24.4	0.75	18.2
7/8	1.14	26.3	1.01	26.6

Table 8. Stream Gaging Results at Lake Warner Outlet

Date (1977)	State (ft)	X-Section Area (sq ft)	Average Velocity (fps)	Flow (cfs)
2/2	0.46	17.5	0.44	7.7
3/2	0.92	33.2	2.08	69
3/11	1.2	42.5	3.23	137
4/13	0.79	28.2	1.91	54
4/20	0.67	24.4	1.43	35
4/29	0.88	32.4	2.22	72
5/25	0.67	23.8	1.31	31
6/2	0.60	23.4	1.24	29
6/6	0.53	19.6	0.77	15
6/14	0.46	18.5	0.60	11

APPENDIX C

DOCUMENTATION OF PHOSPHORUS MODEL

Table 9. Phosphorus Model Input Data Description

Card Group	Format	Card Columns	Description	Variable Name	Unit
1			OPTIONS		
	I10	1-10	Option Index for Snow and DiGiano Model (IOPT = 1) or New Model (IOPT = 0)	IOPT	
2			PHOSPHORUS PARAMETERS		
	F10.3	1-10	Initial Lake P Concentration	PL	µg/ℓ
	F10.3	11-20	Initial Interstitial Sediment P Concentration	PI	µg/ℓ
	F10.3	21-30	Influent P Concentration	PIN	µg/ℓ
	F10.3	31-40	Initial Dry Weight Sediment P Concentration	PS	mg/g
3			RATE COEFFICIENTS		
	F10.5	1-10	P Release Rate Coefficient K_1	RELES	m/day
	F10.5	11-20	P Sedimentation Coefficient K_2	RSED	1/day
	F10.5	21-30	P Desorption Coefficient K_3 (NOTE: only read if IOPT = 0)	U	1/day
4			PHYSICAL PARAMETERS		
	F15.2	1-15	Lake Surface Area	ALAKE	meter^2
	F15.2	16-30	Lake Volume	VLAKE	meter^3
	F10.3	31-40	Lake Detention Time	DETT	days

Table 9. Phosphorus Model Input Data Description (Continued)

Card Group	Format	Card Columns	Description	Variable Name	Unit
5			SEDIMENT PARAMETERS		
	F10.3	1-10	Sediemnt Porosity	POR	
	F10.3	11-20	Sediment Reactive Depth	DREACT	meters
	F10.3	21-30	Fraction Exchangeable P	XCHANG	
6			SEDIMENT PARAMETERS (IF IOPT = 1 ONLY)		
	F10.6	1-10	Slope Coefficient (PI = EM*PS+BE)	EM	
	F10.3	11-20	Ordinate Coefficient	BE	
7			TIME PARAMETERS		
	F10.3	1-10	Total Time of Simulation	TEND	days
	F10.3	11-20	Simulation Time Step	DELTAT	days
	F10.3	21-30	Time Step for Printout	TOUT	days

Table 10. Phosphorus Model Program Listing

```
C      PHOSPHORUS MODEL OF LAKE RECOVERY
C      OPEN LOGICAL UNITS
         N5 = 1
         N6 = 2
         CALL INOPEN(5, N5, 'DAT', 'INPUT FILE?', 11, ERROR)
         CALL PROPEN(5, N6, 'LST', 'OUTPUT FILE?', 12, ERROR)
         IF(ERROR) CALL EXIT
C
C      READ INPUT DATA
C
         READ(N5, 14) IOPT
         READ(N5, 15) PL, PI, PIN, PS
         IF(IOPT.EQ.0) READ(N5, 16) RELES, RSED, U
         IF(IOPT.EQ.1) READ(N5, 16) RELES, RSED
         READ(N5, 20) ALAKE, VLAKE, DETT
         READ(N5, 15) POR, DREACT, XCHANG
         READ(N5, 17) EM, BE
         READ(N5, 15) TEND, DELTAT, TOUT
   14    FORMAT(8I10)
   15    FORMAT(8F10.3)
   16    FORMAT(8F10.5)
   17    FORMAT(F10.6, 7F10.3)
   20    FORMAT(2F15.2, F10.3)
C
C      ECHO PRINT INPUT DATA
C
         WRITE(N5,19) IOPT
   19    FORMAT(10(/), 20X, 'OPTION FOR SNOW (1) OR BINGHAM (0)
        1MODEL = ', I2, //)
         WRITE(N6,21) PL, PI, PIN, PS
   21    FORMAT(20X, 'INITIAL LAKE P CONC. = ', F10.3, ' UG/L',
        1/, 20X, 'INTERSTITIAL P CONC. = ', F10.3, ' UG/L', /, 20X, 'INF
        2LUENT P CONC. = ', F10.3, ' UG/L', /, 20X, 'SEDIMENT P CONC. = ',
        3F10.3, ' MG/G', //)
         IF(IOPT.EQ.0) WRITE(N6,22) RELES, RSED, U
   22    FORMAT(20X, 'RATE OF RELEASE = ', F10.3, ' M/DAY', /, 20X, 'R
        1ATE OF SEDIMENTATION = ', F10.3, ' /DAY', /, 20X, 'RATE OF DESORP
        2TION = ', F10.5, ' /DAY', //)
         IF(IOPT.EQ.1) WRITE(N5,222) RELES, RSED
  222    FORMAT(20X, 'RATE OF RELEASE = ', F10.3, ' M/DAY', /, 20X, 'R
        1ATE OF SEDIMENTATION = ', F10.3, ' /DAY', //)
         WRITE(N6,23) ALAKE, VLAKE, DETT
   23    FORMAT(20X, 'LAKE AREA = ', F15.2, ' M2', /, 20X, 'LAKE VOLUME
        1 = ', F15.2, ' M3', /, 20X, 'LAKE DETENTION TIME = ', F10.3, ' DA
        2YS', //)
         WRITE(N6,24) POR, DREACT, XCHANG
   24    FORMAT(20X, 'SEDIMENT POROSITY = ', F10.3, /, 20X, 'SEDIMENT R
        1EACTIVE DEPTH = ', F10.3, ' M', /, 20X, 'FRACTION EXCHANGEABLE P
        2= ', F10.3, //)
         IF(IOPT.EQ.1) WRITE(N5,25) EM, BE
   25    FORMAT(20X, 'CONSTITUTIVE RELATIONSHIP BETWEEN SOLID AND INTER
        1STITIAL PHOSPHORUS: PI = M * PS + B', /, 30X, 'M = ', F10.6, /,
        230X, 'B = ', F10.3, //)
         WRITE(N6,26) TEND, DELTAT, TOUT
   26    FORMAT(20X, 'TIME OF SIMULATION = ', F10.3, ' DAYS', /, 20X, '
        1SIMULATION TIME STEP = ', F10.3, ' DAYS', /, 20X, 'OUTPUT PRINTE
        2D EVERY ', F10.3, ' TIME STEPS', 5(/))
```

Table 10 (cont.) Phosphorus Model Program Listing

```
C     WRITE HEADING
C
C
        WRITE(N6, 27)
   27   FORMAT(10X, '      TIME      PLAKE     PINTER        PSED',
    1       /, 10X, '     (DAYS)    (MG/L)    (MG/L)       (UG/L)
     2      (MG/G)')
C
C      INITIALIZE
C
        T = DELTAT
        TCHECK = TOUT
        PS = PS * XCHANG * 1000000./2.5
        PINIT = PL
   30   CONTINUE
C
C      CALCULATE PSNEW THROUGH MASS BALANCE ON SEDIMENT
C
        IF(IOPT) 33, 33, 34
   33   PSNEW = PS + (RSED * PL * VLAKE/DREACT/ALAKE - U*PS) * DELTAT
        GO TO 35
   34   PSNEW = PS + (RSED * PL * VLAKE / DREACT / ALAKE - POR * RELES
    1 * (PI - PL) / DREACT) * DELTAT
C
C      CALCULATE INTERSTITIAL VALUE BY MASS BALANCE OR BY LINEAR RELATIONSHI
C
   35   IF(IOPT) 37, 37, 38
   37   PINEW = PI + (U*PS - POR*RELES *(PI-PL)/DREACT)*DELTAT/POR
        GO TO 39
   38   PINEW = (EM * PSNEW + BE) / POR
C
C      CALCULATE NEW LAKE CONCENTRATION BY MASS BALANCE ON LAKE WATER
C
   39   PLNEW = (PIN + POR * ALAKE * RELES * PINEW * DETT / VLAKE) /
    1(1. + POR * ALAKE * RELES * DETT / VLAKE + BSED * DETT)
        A = EXP((-1. / DETT - POR * ALAKE * RELES / VLAKE - BSED) * T)
        PLNEW = PLNEW * (1.-A) + PINIT * A
C
C      NEW TIME STEP
C
        PS = PSNEW
        PSDRY = PS / XCHANG * 2.5/1000000.
        PI = PINEW
        PL = PLNEW
C
C      WRITE OUTPUT AT DESIRED TIME STEPS
C
        IF(T-TOUT*DELTAT) 65, 40, 65
   40   WRITE(N6, 55) T, PL, PI, PS, PSDRY
   55   FORMAT(10X, F10.0, 2F10.3, F15.3, F10.3)
        TOUT = TOUT + TCHECK
   65   T = T + DELTAT
        IF(TEND - T) 75, 30, 30
   75   CONTINUE
        CALL EXIT
        END
```